LONDON PENETRATION DEPTH, PINNING FREQUENCY AND PARA-MEISSNER EFFECT IN SUPERCONDUCTORS

A THESIS
SUBMITTED TO THE
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BY
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JULY, 1998

To my parents

DECLARATION

I here by declare that the work embodied in this thesis has been carried out by me under the supervision of Prof. K. N. Shrivastava, in the school of physics, University of Hyderabad, and the same has not been submitted at any other university.

Place: Hyderabad

Date: 30 July 1998

(Pijush Kanti Ghosh)

CERTIFICATE

Certified that the work embodied in this thesis entitled "LONDON PENETRATION DEPTH, PINNING FREQUENCY AND PARA-MEISSNER EFFECT IN SUPERCONDUCTORS", has been carried out by Mr. Pijush Kanti Ghosh for the full period prescribed under Ph.D ordinances of the University, under my supervision and the same has not been submitted for the award of research degree of any other University.

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CONTENTS

Synopsis

Thesis by	: Pijush Kanti Ghosh
Title of the thesis	: London penetration depth, pinning frequency and para-Meissner effect in superconductors
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This thesis contains seven chapters. In chapter-I, a general review of superconductivity has been given. Occurrence of superconductivity, Meissner effect and the para-Meissner effect have been reviewed. London penetration depth has been discussed. Concept of flux motion has been reviewed. The high as well as low T_c superconductors are discussed in this chapter.

In chapter-II, effect of current loop sizes for the para-Meissner effect has discussed. We find that there is a range of current-loop sizes and a range of temperatures under which the para-Meissner effect is predicted. When the phase ϕ/ϕ_0 of Josephson Hamiltonian varies in a certain range, the magnetization becomes positive. In genaral the magnetization can be both positive as well as negative with zero resistivity in all phases. The susceptibility as a function of temperature at small magnetic fields is explained on the basis of Josephson interaction. The transition temperature of the para-Meissner effect, T_{pM}, is different from that of the Meissner effect, $T_c > T_{pM}$. The experimental mesurements of the magnetization of $Tl_2CaBa_2Cu_2O_8$ at low fields are in agreement with the theoretical predictions.

In chapter-III, viscous damping effect on the magnetic penetration depth in superconducting $ErNi_2B_2C$ is given. The equation of motion of a fluxiod with viscous forces is solved to calculate the effective London penetration depth which is found to vary as the square root of the magnetic field. The London penetration depth as a function of magnetic field in single crystals of $ErNi_2B_2C$ is in resonable agreement with the theory.

In chapter-IV, interpretation of the magnetic field dependence of the penetration depth in superconductors is given. The penetration depth for a superconductor containing oscillating vortices moving in a viscous medium is calculated. For large viscosity it depends on the square root of the field. When the viscosity is zero, it becomes linear in magnetic field. The magnetic field dependence for a viscous medium agrees with that measured in $ErNi_2B_2C$ and that for a nonviscous medium agrees with that in $NbSe_2$ and $YBa_2Cu_3O_7$. When quantized phase of the Josephson current is considered, the penetration depth shows oscillations as a function of magnetic field. Such an oscillatory dependence is indicated in the experimental measurements of muon spin rotation in $CeRu_2$. The Josephson voltage depends linearly on the vortex velocity, which becomes complex so that there is a possibility to switch from one velocity to another. Voltage measurements along the c-axis in $Bi_2Sr_2CaCu_2O_{8+y}$ appear to jump from one velocity to another just as found by theoretical calculations.

In chapter-V, soft vortices in type-II superconductors: YNi_2B_2C and $ErNi_2B_2C$, has been described. We show that the apex angle of the rhombic cell of YNi_2B_2C decreases as a function of increasing magnetic field upto a certain constant field, $B_0 \simeq 1130 \pm 70$ Oe. This dependence is consistant with the mean field theory of vortices. In $ErNi_2B_2C$, the magnetic penetration depth, also exhibits a soft vortex type behaviour below $B_0 \simeq 250 \pm 10$ Gauss. The penetration depth is found to have a component which varies as $B^{1/2}[1-(B/B_0)]^{-1/4}$ as predicted by the mean field theory and shows divergence at B_0. In both the samples, the theoretical predictions are in accord with the experimental measurements. This is the first time that London penetration depth associated with a soft vortex has been reported.

In chapter-VI, soft vortex in $TmNi_2B_2C$ superconductor has been described.The structural distortions, magnetism and superconductivity coexist in single crystals of $TmNi_2B_2C$. It is antiferromagnetic below $T_N \simeq 1.5$ K and superconducting below $T_c \simeq 10.6$ K. We present here a neutron diffraction study which shows that the apex angle β never reaches the value of 90^0 belonging to the square lattice because of the onset of a phenomenon precursor to antiferromagnetism. At some field such as 3 K Oe upon cooling below 4 K, instead of increasing the apex angle starts reducing. The angle tends to zero at the Neel temperature and exhibits a mean field exponent of 1/2. Similarly, upon warming above 4 K, the angle becomes soft at the superconducting transition temperature, T_c. The upper critical field is highly anisotropic at low temperatures and the anisotropy indicates that the gap may exhibit a node. At elevated temperature, 27 K, the upper critical field becomes isotropic as if the gap belongs to s-wave. Therefore the gap appears to have complex symmetry. The change in London penetration depth as a function of temperature has been deduced from the intensity of the neutron diffraction. This measurement also shows node like dependence at low temperatures and s-wave at high temperatures.

In chapter-VII, measurement of pinning frequency of a superconductor by a new method has described. The surface impedence of a system containing viscous damping and oscillating vortices is calculated, of which in the limit of zero vortex mass, the real part is found to vary as $\omega^2(\omega^2+\omega_p^2)^{-1}$ where ω_p is the pinning frequency.

For finite mass of a vortex, a characteristic frequency, ω_r is found. For $\omega_p < \omega_r$ the real part of the resistivity shows peaks and for $\omega_p > \omega_r$ it oscillates. For zero mass of the vortex, the experimental measurements of the surface resistance of 2H-$NbSe_2$ are found to be in reasonable agreement with the theory which is used to measure the pinning frequency, $\omega_p \simeq 145.7$MHz.

Chapter-II of the thesis has been published in "Physica C **294** (1998) 243-248". Chapter-III has been published in "J.Phys.: Condens. Matter **9** (1997) L663-L666". Chapter-V has been accepted for publication in "Physica C".

Chapter-I

Introduction

In this chapter a general review of superconductivity is given. Occurrence of superconductivity, Meissner effect and para-Meissner effect has been reviewed. London penetration depth has been discussed. Concept of flux motion is reviewed. Low T_c as well as high T_c superconductors have been discussed.

One of the importent properties of the materials is their electrical resistivity. Physicists have considerable interest to study the variation of electrical resistivity with temperature. It had been known for many years that the resistance of metals falls when they are cooled below room temperature but it was not known that what limiting value the resistance would approach if the temperature were reduced towards 0 K.

Kamerlingh Onnes [1] at the University of Leiden took such a study of the variation of electrical resistance of metals with temperature. He while experimenting with platinum and gold, found that resistance decreases when cooled but it depends on the purity of the specimen. So he took pure mercury as the sample. In 1911 surprisingly he observed that at a temperature about 4.2 K the resistance of mercury dropped abruptly to a value experimentally undetectable. Furthermore, this phenomenon occured even if mercury was quite impure. He called this remarkable phenomenon as superconductivity and the temperature at which it occured, the critical temperatue (T_c).

This critical temperature (also called transition temperature) varies from metal to metal. Not only metals, a numbers of alloys also have been found to become superconducting at low temperatures. The materials which exhibits superconductivity. are called superconductors.

The disappearence of dc resistance within the critical temperature T_c can be understood by persistent current measurements in a superconducting ring. File and Mills [2] studied the decay of supercurrents in a solenoid by using precision nuclear magnetic resonance method. It can be concluded from the result that there will be no change in field produced by the supercurrent in 10^{10} years. Quinn and Ittner [3] have calculated the resistance of a superconducting thin film tube by measuring the time decay of the current circulating in the tube. They calculated the upper limit of dc resistivity as 3.6×10^{-23} Ohm-cm.

Magnetic properties

Below the transition temperature T$_c$, superconducting behaviour can be quenched and normal conductivity restored by the application of strong external magnetic field. The field at which superconductivity is destroyed is called critical field (H$_c$). The value of the critical field decreases with increasing temperature. The approximate relation between H$_c$ and T$_c$ is

$$H_c(T) \simeq H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right] \tag{1}$$

where $H_c(0) = H_c$ at T=0K. A diagram of magnetic field vs. temperature is called phase diagram. A phase diagram for a superconductor is shown in the figure. 1

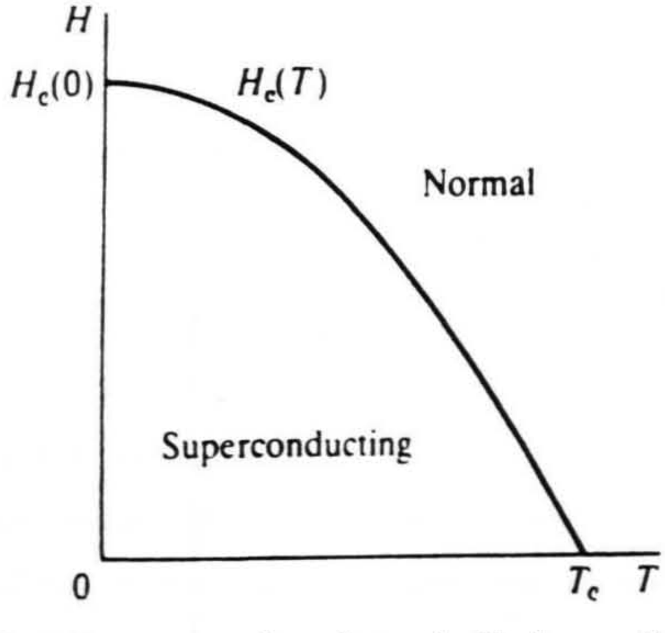

Fig.1 Temperature dependence of critical magnetic field.

Meissner effect

In 1933 Meissner and Ochsenfeld [4] found that when a superconducting specimen cooled below the transition temperature (T$_c$) in the presence of an external magnetic field, the magnetic flux is expelled from the interior of the specimen. This phenomenon is called

Meissner effect. This phenomenon is true for the case when sample is cooled bel ow T$_c$ in the absence of magnetic field and then if magnetic field is applied also. That is to say, inside a superconductor we always have

$$B = 0 \tag{2}$$

where B is the magnetic flux density. Now we have the relation

$$B = H_a + 4\pi M \tag{3}$$

where H_a is the applied magnetic field and M is the magnetization. Now according to the condition (2) equation (3) becomes

$$H_a = -4\pi M \tag{4}$$

Therefore susceptibility becomes

$$\chi = \frac{M}{H_a} = -\frac{1}{4\pi} \tag{5}$$

Thus superconductors shows the property of perfect diamagnetism.

Type-I and Type-II superconductors

Pure specimen of many materials shows complete Meissner effect, that means below the critical temperature flux inside the superconducting specimen is zero. Thus there is only one value of critical field (H$_c$) available for a particular temperature T, ($T < T_c$). These materials are called Type-I superconductors.

Transition metals and alloys show different behaviour, there are two critical fields H$_{c1}$ and H$_{c2}$ available for a perticular temperature T, ($T < T_c$). Between H$_{c1}$ and H$_{c2}$. Meissner effect is said to be incomplete, specimen is threaded by magnetic flux lines, these are called vortices. These superconductors are known as Type-II superconductors. Here one point we can mention that all high-T_c superconductors are Type-II superconductors.

Para-Meissner effect

The field cooling Meissner effect of some ceramic high-T_c samples was reported [5] to remain incomplete at $H << H_{c1}$. The flux expulsion for these samples is of the order of $\frac{1}{3}$ of a zero field cooling sample. This incompleteness was explained by flux pinning and anisotropy of the London penetration depth. It is quite common to find values of less than $-1/4\pi$ due to imperfections in the samples. Shrivastava and Braunisch et al. [6] reported that in certain Bi-based high-T_c superconductors a paramagnetic magnetisation is found in the field-cooling mode, below a field of order 1 Oe. Since this magnetization appears spontaneously, this phenomenon is called as paramagnetic Meissner effect or Para-Meissner effect (PME).

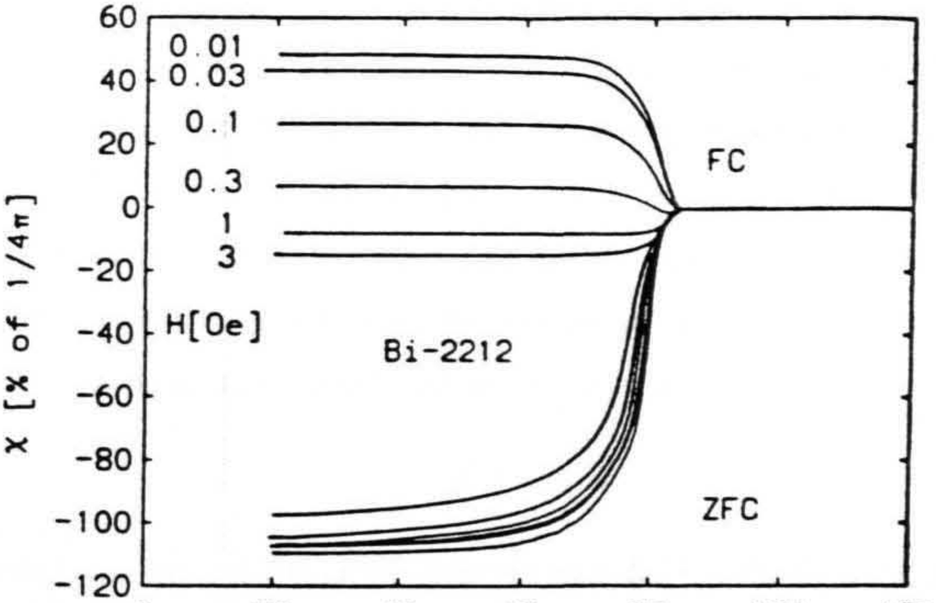

Fig.2 Zero field cooled (ZFC) and field cooled (FC) signals of a ceramic Bi-2:2:1:2 ($Bi_2Sr_2CaCu_2O_{8-x}$) sample exhibiting the para-Meissner effect [6].

Heinzel et al. [7] experimentally confirmed the occurrence of PME by the measurement of the second harmonic of the magnetic ac-susceptibility. This PME can be explained on the basis of π-junctions between the grains of the granular high-T$_c$ superconductors, which was thought to arise as a consequence of d-wave symmetry. Recently Araujo-Moreira et al. [8] reported the occurrence of PME without the presence of π-junctions. Our work on current loop sizes of PME is given in chapter-II.

The London Equations

London and London [9] advanced a phenomenological theory of the electromagnetic behaviour of superconductors. Their theory based on the two fluid model.

According to this two fluid model, in a superconducting material a finite fraction of electrons condenses into 'superfluid' which extends all over the volume of the specimen. At absolute zero temperature the condensation is complete and all the electrons participate in forming the superfluid. As the temperature increases, fraction of electrons evaporate from the condensate and form normal fluid. As the temperature approaches critical value T$_c$, the fraction of electrons remaining in the superfluid tends to zero.

If n_s and n_n be the superelectron and normal electron densities respectively, then $n_s+n_n = n$, where n is the avarage number of electrons per unit volume.

Below T_c, the supercurrent density J_s can be written as

$$J_s = -en_sv_s \tag{6}$$

where v_s is the drift velocity of the super electrons and e is the electronic charge.

when an electric field E is applied, then the force equation can be written as

$$m\frac{dv_s}{dt} = -eE \tag{7}$$

Now from equation (6), with the help of equation (7) we can write,

$$\frac{dJ_s}{dt} = \frac{e^2 n_s}{m} E \tag{8}$$

This is known as first London equation. This equation shows steady current is possible in the absence of electric field.

Operating curl on both sides of equation (8) and with the help of Maxwell's equation we can get,

$$\nabla \times J_s = -\frac{n_s e^2}{mc} B \tag{9}$$

This is known as second **London** equation which explains Meissner effect. In terms of vector potential A, equation (9) can be written as

$$J_s = -\frac{c}{4\pi \lambda_L^2} A \tag{10}$$

where $\lambda_L = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{\frac{1}{2}}$ is known as London penetration depth. It may be defined as the distance where the magnetic field reduces to $\frac{1}{e}$ times its bound~~e~~ary value. The expression for λ_L can be written in more appealing form as

$$\lambda_L = \left(\frac{mc^2}{4\pi n_s e^{*2}}\right)^{\frac{1}{2}} \tag{11}$$

Here $e^* = 2e$ for superconducting state revealed from from flux quantisation measurements. This λ_L plays an important role in characterizing a superconductor.

Temperature and magnetic field dependence of London penetration depth

According to the London theory λ (here λ denotes London penetration depth) is independent of temperature and magnetic field, but experimentally these dependence is found. The temperature and magnetic field dependence of λ is not well understood till today. Numbers of researchers are trying to find out exact dependence of λ with temperature as well as with magnetic field.

If we take the temperature dependence of super-electron density then from Gorter-Casimir model [10] we can write,

$$n_s(T) = n\left[1 - \left(\frac{T}{T_c}\right)^4\right] \tag{12}$$

so that temperature dependence of λ comes as

$$\lambda(T) = \lambda(0)\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-\frac{1}{2}} \tag{13}$$

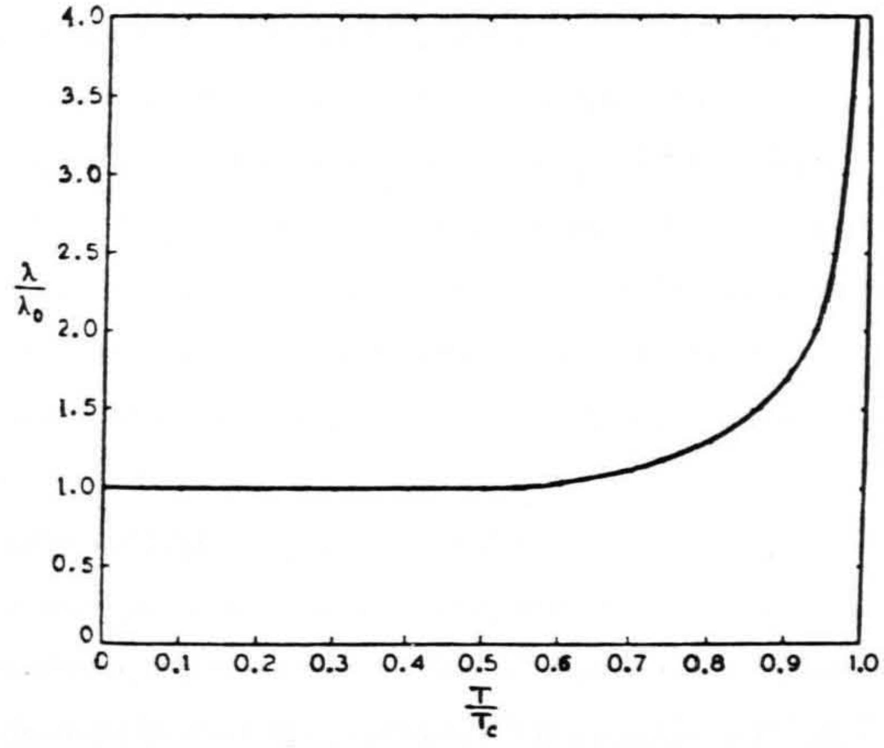

Fig.3 Temperature dependence of the superconducting penetration depth according to Gorter-Casimir two fluid model [11].

Annett et al. [12] have shown that all the possible non s-wave singlet pairing states of a superconductor with tetragonal or orthorhombic symmetry and a Fermi surface that has spherical or cylindrical topology have line nodes in the gap that give rise to a linear temperature dependence in the penetration depth, $\Delta\lambda(T) \propto T$ unless scattering or Fermi-liquid corrections are import~~e~~ant, ~~W~~where $\Delta\lambda(T) = \lambda(T) - \lambda(0)$. They predicted that kinetic measurements can give a quadratic temperature dependence. In s-wave BCS theory [13] for a spherical Fermi surface the temperature dependence is given by,

$$\frac{\Delta\lambda(T)}{\lambda(0)} \sim 3.33 \left(\frac{T_c}{T}\right)^{\frac{1}{2}} exp\left(-1.76\frac{T_c}{T}\right) \tag{14}$$

The exponential term is a consequence of the energy gap Δ, which takes on the value $\Delta/k_B T_c = 1.76$ in weak coupling BCS theory. Prohammer and Carbotte [14] calculated the London penetration depth for d-wave superconductors stabilized by antiferromagnetic spin fluctuations. They found $\lambda(T) \sim T$ at low temperatures, but impurity scattering as in the case of p-wave order parameters changes T toT^2 dependence.

Hardy et al. [15] have observed a linear temperature dependence of $\Delta\lambda(T)$ below 20 K in single crystal $YBa_2Cu_3O_{6.95}$ which is very different from that expected for s-wave BCS superconductivity. They predicted that the strong linear dependence to be the characteristic of the pure system and that its apparent absence in thin films and some crystals is due to the presence of defects.

Kosztin and Legget [16] have shown that at very low temperatures nonlocality may play an important role in the electromagnetic response of a d-wave (unconventional) superconductor, which leads to $\Delta\lambda(T) \propto T^2$ dependence. They predicted that this T^2 dependence can be observable experimentally in nominally clean high-T_c superconductors below a crossover temperature $T^* \sim 1K$ but above this crossover temperature T^* and below T_c, $\lambda(T)$ has the well known linear T dependence. The penetration depth in a mixed wave superconductor has been calculated [17] and it was shown that at low temperature the ground state has $d_{x^2-y^2}$ symmetry which at higher temperature becomes s-wave type.

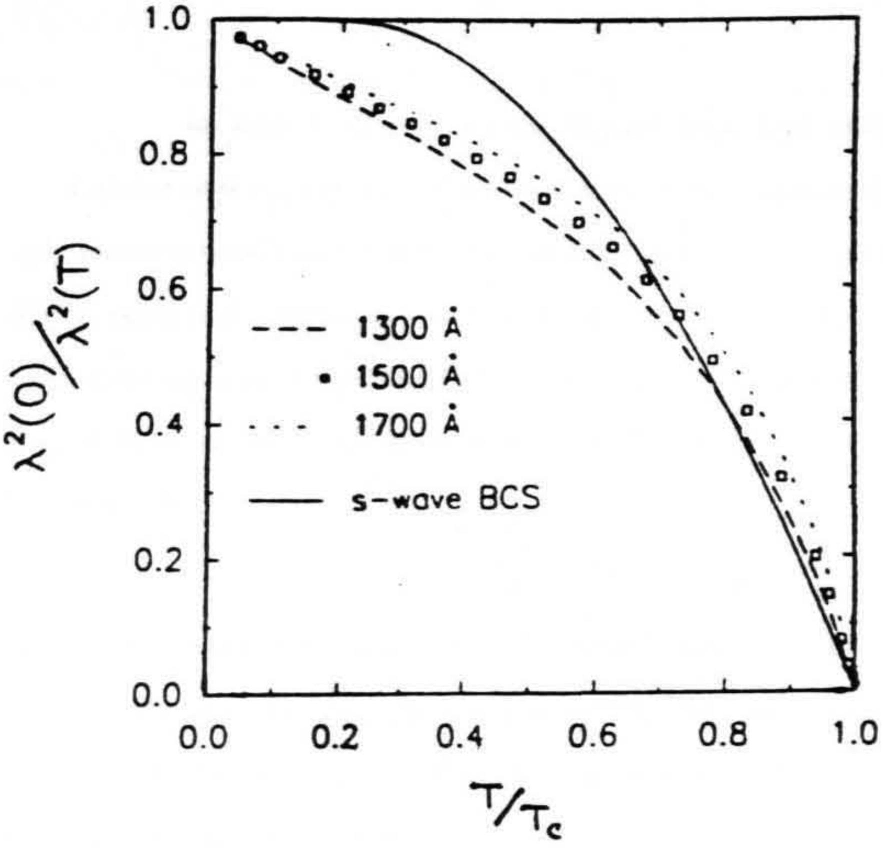

Fig.4 Variation of $[\lambda(0)/\lambda(T)]^2$ with temperature for single crystal $YBa_2Cu_3O_{6.95}$ for representative range of $\lambda(0)$: 1300, 1500 and 1700 Å. The overall shape shows the strong linear behavior at low temperatures [15].

It was thought that, the penetration depth λ, of a magnetic field into a superconductor should be regarded not as independent of field strength at constant temperature but rather as increasing with field. Pippard [18] while studying experimentally the magnetic field variation of superconducting penetration depth found that the variation is considerably small. The result of the experiment suggested the existance of long-range order in the superconducting state over a distance of 10^{-4}cm or more.

Our work on variation of penetration depth with magnetic field is given in different chapters of the thesis.

Flux motion

It is known that for high-T_c superconductors H_{c2} can be as high as 10^6 Oe or higher. The basic criteria for making a superconducting magnet is that superconducting material must not only have a critical field substantially higher than the field to be produced but it must be able to carry high current in that field without resistance. The resistanceless current in a homogeneous type-II superconductor is limited to that value which just produces the field H_{c1} of the superconductor at its surface. For a wire of radius a this condition is given by Silsbee's rule

$$\frac{2I}{a} = H_{c1} \tag{15}$$

At H_{c1} the superconductor enters into the mixed state and sample contains both the transport current and the magnetic flux threading through the bulk of the superconductor. Due to their coexistance magnetic flux exerts Lorentz force on the current carriers. This force per unit volume is given by,

$$F = J \times \frac{B}{c} \tag{16}$$

where J is the current density and B is the average magnetic induction. To this force there is an equal and opposite reacting force which acts on magnetic flux lines referred to as Lorentz driving force. This driving force which acts on a single vortex can be written as

$$f_L = J \times \frac{\phi_0}{c} \tag{17}$$

where ϕ_0 is the flux quantum. Because of this force, flux lines tend to move transverse to the current. In a homogeneous medium there is no counteracting force which results in which the vortex lines are driven into motion. This vortex motion gives rise to resistance which is not of practical interest.

Clearly to carry high current without resistance the vortex lines must be pinned so that their motion is inhibited. This can be done by introducing in the material various types of inhomogeneities, which may include lattice defects such as dislocations, precipitates, grain boundaries etc. In hard superconductors inhomogeneities offer pinning force counteracting the driving force and a static non-uniform vortex distribution becomes permissible, provided $f_L < f_p$, where f_p is the maximum pinning force acting on each vortex. Thus no flux motion is to be expected until $f_L > f_p$. Anderson [19] pointed out that this would be so only at 0 K. At any finite temperature $T < T_c$, thermal activation aided by the driving force may cause the pinned vortices to overcome the barrier and move, even when $f_L < f_p$.This behaviour is given the name flux creep.

When $f_L > f_p$ the creep-like motion of vortex lines changes to a highly dissipative one chacterized by a viscous flow. The motion of the vortices limited by viscous drag referred as flux flow. If the vortex line move with a velocity v_L, the force equation can be written as

$$J\frac{\phi_0}{c} = \eta v_L \tag{18}$$

where η is the co-efficient of viscosity. This flux flow usually gives a flow resistivity ρ_f,

$$\rho_f = \frac{E}{J} = B\frac{\phi_0}{\eta c^2} \tag{19}$$

A general model of this problem has been developed by Bardeen and Stephen [20]. Our work on pinning frequency is given in different chapter.

Progress of high-T_c

Since 1911, when K. Onnes discovered superconductivity in mercury at 4.2 K, the highest observed values of T_c gradually moved upward. But this increament was not so considerable. In 1973 Gavaler [21] observed that sputtered films of Nb_3Ge began to become superconducting at 22.3 K and this was soon pushed up to 23.2 K by altering the sputtering conditions slightly. Inspite of great efforts to increase this limit further, it stood as the record untill 1986. The breakthrough of high temperature superconductivity came when Bednorz and Müller [22] reported that a lanthanum barium copper oxide began its superconducting transition as it was cooled below 35 K. All of the subsequent work proved that there were high temperature superconductors.

In rapid succession and in many laboratories the barium was substituted by strontium and calcium and the transition temperature was raised to nearly 40 K for the material $La_{1.85}Sr_{0.15}CuO_4$. Subsequent effort to raise T_c was carried out by many scientist. M.K.Wu et al.[23] reported first the material capable of becoming superconducting in liquid nitrogen; it turned out to be $YBa_2Cu_3O_{7-\delta}$ with T_c about 93 K. Thus with the discovery of high-T_c superconductors they can be operated with liquid nitr$\overset{o}{}$gen which is much less expensive and easy to handle compared to expensive liquid helium. The highest T_c for these series of compounds (Oxide superconductors) known today is 148 K. → which (Ref?)

Applications

The most useful application of superconductors is to make superconducting magnets (solenoid) which can supply steady fields of over 100,000 G without dissipation of energy because of the resistanceless persistent current. A comparable field produced by a water-cooled copper solenoid which dissipate several megawatts of power, with attendant cooling problems and it would not have the essentially infinite stability of the superconducting magnet. Superconducting magnets find applications in many areas in technology. Beside this, superconductors used in making memories, logic gates in computers; voltage standard, radiation detectors; and in biomedical, geophysical applications.

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Chapter-II

Effect of current-loop sizes on the para-Meissner effect in superconductors

We find that there is a range of current-loop sizes and a range of temperatures under which the para-Meissner effect is predicted. When the phase ϕ/ϕ_o of the Josephson Hamiltonian varies in a certain range the magnetization becomes positive. In general the magnetization can be both positive as well as negative with zero resistivity in all phases. The susceptibility as a function of temperature at small magnetic fields is explained on the basis of Josephson interaction. The transition temperature of the para-Meissner effect, T_{pM}, is different from that of the Meissner effect, $T_c > T_{pM}$. The experimental measurements of the magnetization of $Tl_2CaBa_2Cu_2O_8$ at low fields are in agreement with the theoretical predictions.

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Introduction

The susceptibility of a superconductor below the transition temperature, T_c, becomes negative approaching the value of $-\frac{1}{4\pi}$ as the temperature approaches zero due to the perfect diamagnetism. For the externally applied field below a critical value, H_{c1}, the magnetic field inside the superconductor is zero so that the lines of force are expelled from the sample. This phenomenon is called the Meissner effect. Recently, it has been reported [1,2] that at low magnetic fields, in field cooled (FC) samples for $T < T_c$ and $H < H_{c1}$, the magnetization becomes positive and at elevated fields does not approach the value of $-\frac{1}{4\pi}$ due to flux trapping. This phenomenon is called the para-Meissner effect (PME). It was thought [2] that a paramagnetic term may be added to the susceptibility and hence similar observations were called the paramagnetic Meissner effect [3]. When the top and bottom surfaces of the superconducting Nb disks were abraded, the PME was reduced and even eliminated while abrading the circumferential edge had a minimal effect on the PME. It was reported that the surface microstructure plays a key role in the formation and the size of the PME [4]. If the phase of the Josephson current is shifted by π, it gives rise to the change in sign of the free energy and hence it may provide the explanation for the change in sign of the magnetization [5]. Independent of the PME, there has been considerable interest in the π-junctions. In a junction if one of the elements is made of a s-wave superconductor and another of a d-wave superconductor seperated by an insulator, then a change in phase of the current is expectd as it passes through the junction [6,7].

In this chapter, we report that there are current-loop size restrictions on the observation of the para- Meissner effect. Similarly there is a temperature range in which para-Meissner effect is visible so that the critical temperature of the PME is lower than that of the superconductor. In the case of zero-field-cooled (ZFC) samples of $Tl_2CaBa_2Cu_2O_8$ we observe that the relaxation rate of the magnetization varies linearly with the applied field.

Size effect

We assume that the sample consists of a single superconducting current-loop with one Josephson junction. This system is solvable with a few approximations but not for the arbitrary shape of the loop. We are able to find the solution for a linear branch of the loop in which the approximate diameter of the loop is treated as the size. We consider that the inductance of the loop is L and current is J [6]. The flux penetrating the loop is ϕ so that the energy of the loop is given by,

$$\epsilon = \frac{1}{2}LJ^2 - \frac{J\phi_o}{2\pi}\cos\left[\frac{2\pi}{\phi_o}(\phi + LJ)\right] \tag{1}$$

The magnetic field arises in the above due to the flux quantization. $\phi = H.A$ with A as the area of cross section. The magnetization of this energy is given by $\partial\epsilon/\partial H$,

$$M = JA\ \sin\left[\frac{2\pi}{\phi_o}(\phi + LJ)\right] \tag{2}$$

which is positive for

$$H_{o1} > H > H_{o2} \tag{3}$$

and negative for

$$H_{o1} < H < H_{o2} \tag{4}$$

where

$$H_{o1} = \frac{1}{A}\left[\frac{\phi_o}{2}(2n+1) - LJ\right],$$
$$H_{o2} = \frac{1}{A}[n\phi_o - LJ] \tag{5}$$

with n as an integer. The positive values give rise to the para-Meissner effect (PME) whereas the negative values give the Meissner effect.In general, the loop will be of a random shape. If it is treated approximately as a circle, the diameter of the loop can be thought of the size of the loop. In the case of a square loop, the length of the square may be treated as the size. In such a case the problem is integrable along a length,l. We consider that the sample contains only one junction. Therefore,we assume that the loop size is l so that in order to determine ~~the~~ the energy of the sample of length l we multiply (1) by dz and integrate from 0 to l. The distance dependence of the magnetic field is taken to be $\hat{\mathrm{H}} = H\epsilon^{-z/\lambda}$ where λ is the London penetration depth. The magnetization of the loop is assumed to decay as a function of distance as $\epsilon^{-z/\lambda}$ travelling along a sample of length l which may be linear along the length of a square or approximately equal to the diameter of a loop. The magnetization can then be written as

$$M = AJ\int_0^l \epsilon^{-z/\lambda} \sin\left[\frac{2\pi}{\phi_o}(AH\epsilon^{-z/\lambda} + LJ)\right] dz \tag{6}$$

which is integrated to

$$M = \frac{J\phi_o\lambda}{2\pi H}\left[\cos\frac{2\pi}{\phi_o}(AH\epsilon^{-l/\lambda} + LJ) - \cos\frac{2\pi}{\phi_o}(AH + LJ)\right] \tag{7}$$

We define two lengths,

$$l_{o1} = -\lambda\,\ln\left[\{\frac{\phi_o}{2}(2n + \frac{1}{2}) - LJ\}\frac{1}{AH}\right]$$

and

$$l_{o2} = -\lambda\,\ln\left[\{\frac{\phi_o}{2}(2n + \frac{3}{2}) - LJ\}\frac{1}{AH}\right] \tag{8}$$

for the size of the loop. The magnetization given by (7) shows negative sign only within the range of lengths

$$l_{o2} > l > l_{o1}. \tag{9}$$

When the first term of (7) is negative and the second term is small but positive, the magnetization becomes negative. When the angle of the second cosine term in (7) is small

$$\frac{2\pi}{\phi_o}(AH + LJ) \leq (2n + \frac{3}{2})\pi + \frac{\pi}{4}. \tag{10}$$

This is equivalent to a field restriction, $H \leq H_{o3}$, where

$$H_{o3} = \frac{1}{A}\left[\frac{\phi_o}{2}\{2n + \frac{3}{2} + \frac{1}{4}\} - LJ\right] \tag{11}$$

For loops of lengths which do not satisfy (9), the magnetization becomes positive. For $l << \lambda$, $e^{-l/\lambda} \simeq 1 - l/\lambda$ so that the argument of the cosine can be expanded. For small angle the cosine term may also be expanded so that

$$M = \frac{\pi l J H A^2}{\phi_o}\left[2 + \frac{LJ}{AH} - \frac{l}{\lambda}\right] \tag{12}$$

which gives positive (para-Meissner) magnetization. Therefore depending on the loop size, either the Meissner effect or the PME would occur. This prediction is qualitatively in accord with the measurements on Nb disks [4] where surface abrading is found to show an observable effect.

Temperature restriction

The Josephson current varies with the gap of the superconductor, $J = c_1\Delta$. According to the BCS theory the gap varies as $\Delta = \Delta_o(1 - T/T_c)^{1/2}$ so that

$$J = c_1\Delta_o(1 - T/T_c)^{1/2} \tag{13}$$

The current from the first of the equations of (5) is given by

$$J = \frac{\phi_o}{2L}(2n + 1) - \frac{AH_{o1}}{L} \tag{14}$$

so that the limiting field varies with temperature as

$$H_{o1} = \frac{\phi_o}{2A}(2n + 1) - \frac{Lc_1\Delta_o}{A}(1 - T/T_c)^{1/2} \tag{15}$$

This field is positive only if $T > T_{pM}$ where

$$T_{pM} = T_c[1 - \{\phi_o(2n+1)/2Lc_1\Delta_o\}^2] \tag{16}$$

Thus, the Meissner effect occurs only above H_{o1} and above T_{pM}. A new transition temperature is thus generated below which the phase of the current is shifted compared with that in the usual Meissner state. In the new state the magnetization is positive and the resistivity is zero. Thermodynamically, the new state will have its own transition temperature ,T_{pM} and its own law for the variation of susceptibility as a function of temperature, critical field etc. If the current and the magnetic field are treated as parallel to each other in the Meissner phase, then for the fixed direction of the magnetic field, the current is anti-parallel in the para-Meissner phase.

Relaxation rate of the ZFC magnetization

The low field magnetic relaxation has been measured by Mota et al. [8] in $Sr_{0.2}La_{1.8}CuO_4$ within the field range $25 < H < 385$ G . It was found that $-\partial M/\partial lnt \propto H^3$ at 4.2 K. Since the magnet has a residual field, the measurements could not be performed for H< 25G.

For the samples of $Tl_2CaBa_2Cu_2O_8$ of $T_c \simeq 105$ K prepared by the method described elsewhere [9], we show the relaxation rate of the magnetization of the ZFC samples for H < 10 G in Fig.1. It is seen that at low fields, $\partial M/\partial lnt$ is linearly dependent on H where t is the time. In eq.(1) the magnetic field appears only as ϕ in the phase factor from which there is no way of obtaining H^2 type term. However, in between the loops the energy term in the normal material will have the usual square term as,

$$\epsilon_1 = -\frac{H^2}{8\pi} \tag{17}$$

which contributes a term $M = \partial\epsilon_1/\partial H \simeq -H/4\pi$ to the magnetization. Thus $t\partial M/\partial t$

has a term proportional to the field, H in addition to the sine term arising from the phase factor as in (2). For small arguement, $\sin\theta \simeq \theta$, i.e., for small fields (2) also gives a term linear in the field,H. In our experimental situation, field is never less than ϕ_o so that the linear term arising from (2) is not the proper cause of the linear observation and hence (17) is more appropriate.

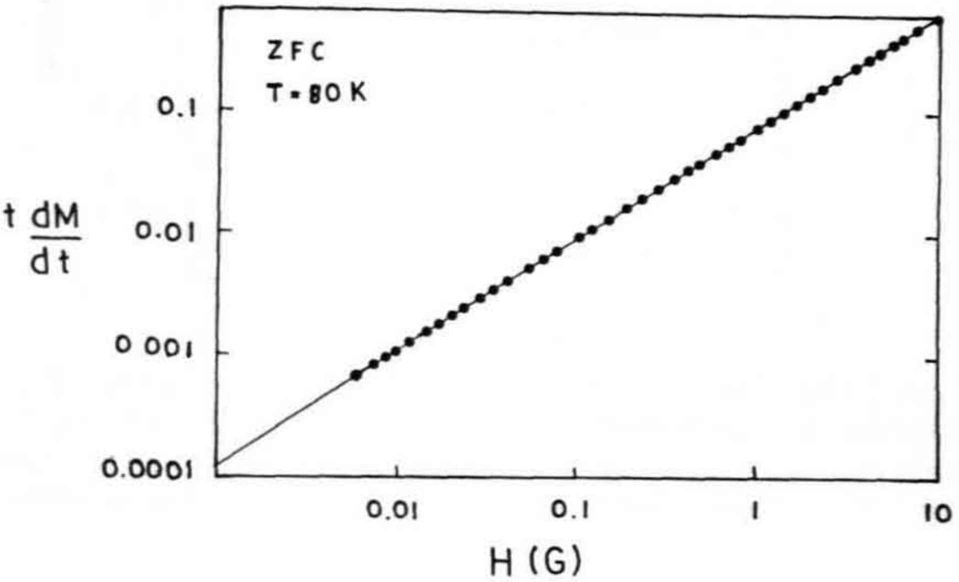

Fig.1 The relaxation rate of the ZFC magnetization of $Tl_2CaBa_2Cu_2O_8$ measured as a function of the applied magnetic field using Mumetal shielded SQUID magnetometer showing that $\partial M/\partial \ln t$ is a linear function of the magnetic field at low fields.

It shows that the system is radiating in a free field. In the microwave absorption this material showed [10] a paramagnetic unstable state below T_c. In Fig.2 we show the experimentally measured value of the magnetization for a FC as well as for a ZFC sample. It is observed that the FC data gives positive whereas ZFC gives negative magnetization. In Fig.3 we show the magnetization as a function of temperature which becomes negative as the field is increased showing that the Meissner effect is recovered at elevated fields.

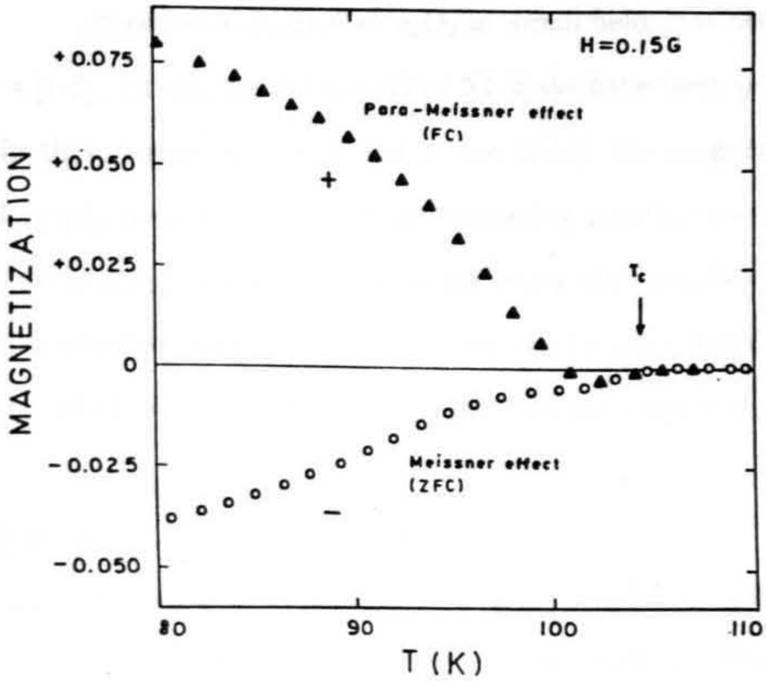

Fig.2 The magnetization as a function of temperature at a fixed field of H = 0.15 G for both the FC and ZFC samples of $Tl_2CaBa_2Cu_2O_8$ showing positive magnetization and hence the para-Meissner effect in the FC samples and negative magnetization corresponding to the Meissner effect in ZFC samples.

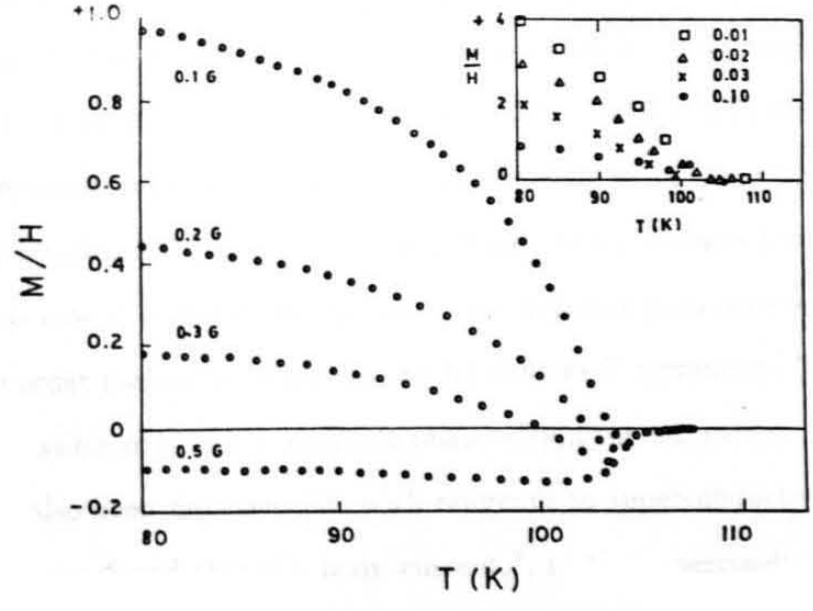

Fig.3 The magnetization as a function of temperature while cooling the samples in a fixed applied magnetic field (FC). There is a small field above which the magnetization becomes negative. The inset shows the same data at low magneic fields. The field inside the Mumetal shield is ~ 1 mG.

The magnetization of $Bi_2Sr_2CaCu_2O_8$ at small fields has been reported by several researchers [1-3]. Similar measurements in Nb disks have been performed by Thompson et al [4]. In these works, it is seen that at low fields, the magnetization which is negative below T_c turns towards positive values becoming positive upon cooling. The change in the sign of the magnetization can be described by the Josephson interaction. The turning point from negative towards positive values can be interpreted to arise from the change in the sign of the cosine term at $\pi/2$ which does not require π-junctions.

Flux Quantum

Buyers and Yang [12] found that the flux is quantized for any value of charge, Σe. There is no particular value of the charge for which flux is quantized. On the basis of experimental observation of Shapiro steps, it can be said that charge of the quasiparticles is 2e in the superconducting state so that the unit of flux becomes, $\phi_o = hc/2e$. Geshkenbein et al.[13] suggest that flux is quantized in units of $\phi_o(n + 1/2)$ where n is an integer so that the difference between any two values is a multiple of ϕ_o only and the minimum value is $\phi_o/2$. For a flux-quantized Lorentz force work, equally spaced absorption lines are found [14] so that the energy is determined by $J\phi(n + 1/2)$ where J is the current. This observation implies that the minimun flux may be $\phi_o/2$. Within the framework of Ginsburg-Landau (GL) model, there is a phase which violates time reversal invarience ,$\mathcal{T}$. In this case it is found [15] that all values different from $\phi_o(n + 1/2)$ can occur only when the order parameter of the GL model violates $\mathcal{T}$ symmetry. Thus values less than $\phi_o/2$ are possible only in a $\mathcal{T}$ violating phase within the GL model. Such a $\mathcal{T}$ breaking B phase has also been discussed [16] with reference to superconductivity in UPt_3. Buyers and Yang have found that the body current,I, in the superconductor is related to the partition function,Z, as

$$I = ck_BT\partial \ln Z/\partial\phi \tag{18}$$

which can be written as

$$E = \frac{I}{c}d\phi = k_BTd\ln Z \tag{19}$$

replacing $d\phi$ by $n\phi_o$ we obtain

$$E_n = \frac{I}{c}n\phi_o \tag{20}$$

so that we obtain equally spaced levels. We replace n by n+1/2. Accordingly $E_n = \frac{I}{c}\phi_o(n+1/2)$, so that energy and current become linearly dependent. Buyers and Yang have also derived an expression for the quadratic dependence of ϕ on the energy. The magnetization is determined by (2)and the susceptibility of the system for L=0 is given by $\partial M/\partial H$ so that

$$\chi = \frac{2\pi A^2 J}{c\phi_o}\cos\left(\frac{2\pi\phi}{\phi_o}\right) \tag{21}$$

The cosine function changes sign at an angle of $\pi/2$ so that if there is a point at which the susceptibility changes sign from positive to negative values, then such a point occurs at $2\pi\phi/\phi_o = \pi/2$ or at $\phi = \phi_o/4$. Thus smallest flux becomes equal to $\phi_o/4$ whereas only $\phi_o/2$ is predicted by the linear theory. The experimental measuremets as well as the theoretical calculations are thus consistent with our interpretation of the phase factor so that the minimum observable flux becomes $\phi_o/4$. In addition to the inductance term, the capacitance term has been considered by Aranjo-Moreira et al. [17]. These authors simulate the in-phase component of the first harmonic which also shows para-Meissner effect consistent with our interpretation [1].

Discussions

The lines of force of the magnetic field are expelled out of the superconducting sample so that the induction field inside the sample is zero, $B_{in} = 0$. According to Maxwell's equations, the $B_{in} = 0$ is completely consistent with the zero resistivity, $\rho = 0$. After the present work, the phase ϕ/ϕ_o of the Josephson Hamiltonian is found to change to $(\phi/\phi_o)[1 + (2n + \frac{1}{2})\pi]$ till $(\phi/\phi_o)[1 + (2n + \frac{3}{2})\pi]$ so that the magnetization can be both positive (para-Meissner) or negative (Meissner) with zero resistivity in all the phases. Hence the para-Meissner phase is superconducting the same way as the Meissner phase is.

In the case of π-junctions, the phase change of π shows that the symmetry is that of the d wave. Thus the positive sign in the magnetization arises from the phase factor.

The atomic paramagnetism arises from the unfilled d-shell of the transition elements such as copper in the present case. The pair current is diamagnetic for all the phases and there are no unpaired single electrons which are paramagnetic.

Hence we conclude that the PME is not caused by atomic paramagnetism. The phases of the two electrons of the Cooper pair are so correlated that there is a change in the phase in going from the s-wave to the d-wave leading to positive (para-Meissner) magnetization. When the field is changed the phase difference between the two electrons changes so that the para-Meissner effect changes to the Meissner effect. If $H_{c1} > H_{o2}$, as well as $H_{c1} > H_{o2}$ the PME region occurs for $0 < H < H_{o1}$ and ME is found for $H_{o1} < H < H_{o2}$ but a second PME region occurs for $H_{o2} < H < H_{c1}$. If $H_{c1} < H_{o2}$,the PME occurs for $0 < H < H_{o1}$, ME occurs for $H_{o1} < H < H_{c1}$ and then a para-Abrikosov region occurs for $H_{c1} < H < H_{o2}$. Just as the London penetration depth and the coherence lengths are given in terms of two critical fields. $\lambda^2 = n\phi_o/H_{c1}$ and $\xi^2 = n\phi_o/H_{c2}$, we find that H_{o1} and H_{o2} should also be associated with lengths,i.e., $\xi_1^2 = n\phi_o/H_{o1}$ and $\xi_2^2 = n\phi_o/H_{o2}$ at which the current changes sign.

The phase difference can also be expressed as the integral $(2\epsilon/\hbar)\oint \varphi dt$ around a closed circuit where φ is evaluated at the centre of the wave packet. This phase is then similar to the Aharanov-Bohm phase [11] except that we have 2e in place of e for the charge of the carriers.

Conclusions

We find that there are only certain size current-loops which give the para- Meissner effect. Similarly, only in a certain temperature range the para- Meissner effect is observable. Out side the range of the junction size and temperature needed for the para- Meissner effect, the magnetization becomes negative. The phase difference between the two electrons of a Cooper pair causes a change in the sign of the magnetization leading to para-Meissner effect. The measurement of the magnetic relaxation in a zero-field cooled (ZFC) sample shows that $\partial M/\partial \ln t$ is a linear function of the magnetic field. The para-Meissner phase appears as a thermodynamic phase with its own critical field and critical temperature.

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Chapter-III

Viscous damping effect on the magnetic penetration depth in superconducting $ErNi_2B_2C$

The equation of motion of a fluxiod with viscous forces is solved to calculate the effective London penetration depth which is found to vary as the square root of the magnetic field. The London penetration depth as a function of magnetic field in single crystals of $ErNi_2B_2C$ are in reasonable agreement with the theory.

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Introduction

The London penetration depth is determined by the constant factor when the diamagnetic critical current is expressed in terms of the vector potential,

$$\mathbf{J} = -\frac{c}{4\pi\lambda_L^2}\mathbf{A}. \tag{1}$$

In such an expression the wave vector dependence of the electron operators has not been considered. The inclusion of the wave vector dependence of the electrons and the suitable summation over such wave vectors requires the use of density of states [1]. Kogen et al [2] have derived the London equations from the microscopic theory which show that the inverse square of the London penetration depth depends on the density of states at the Fermi level per spin. Won and Maki [3] have found that the residual density of states varies as the square root of the magnetic field so that we can conclude that λ_L varies as the inverse one-fourth power of the magnetic field, $\lambda_L \propto B^{-1/4}$. Yip and Sauls [4,5] have considered nonlinear current-velocity equation from which they argued that pair-breaking effect reduces the effective superfluid density and hence penetration depth increases quadratically with the magnetic field, $\lambda_L \propto B^2$. For field parallel to a node, the effective penetration length is predicted to be proportional to the field,$\lambda_{eff} \propto B$. Affleck et al [6] have generalized the London model starting from a Ginzburg-Landau free energy density to consider the coordinate dependence of the order parameter which explains the flux-lattice structures. This work [7] can be interpreted to give rise to a wave vector dependence and hence temperature and field dependence of the effective penetration depth. Kosztin and Leggett [8] have shown that non-local electrodynamics gives rise to a T^2 dependence in the London penetration depth. It has been reported by Eskildsen et al [9] that rf kinetic inductance as measured in a tunnel diode changes the resonant frequency due to changes in the penetration depth. Such a measurement in $ErNi_2B_2C$ shows $\lambda_{eff} \sim \sqrt{B}$ which is not understood theoretically.

In this chapter, we show that fluxiod viscous damping gives rise to an effective London penetration depth which at large fields varies as the square root of the magnetic field. The calculation is in accord with recent neutron reflectivity and inductance measurements of the penetration depth in $ErNi_2B_2C$ as a function of magnetic field up to 2 kOe.

Fluxiod viscous damping

We assume that fluxiods are subject to a harmonic force with the force constant k, the viscous force ηv proportional to the velocity, v and the ordinary fluxiod mass M, multiplied by the acceleration is determined by the quantized flux,

$$M\frac{dv}{dt} + \eta v + kx = \frac{1}{c}J\phi_o \tag{2}$$

The fluxiod velocity has the time dependence of $v = v_o e^{-i\omega t}$ which substituted in (1) above gives,

$$v = \frac{J\phi_o}{c(\eta - i\omega M + ik/\omega)}. \tag{3}$$

The moving fluxiod produces an electric field , $E_\varphi = -\frac{1}{c}vB$, that opposes the current so that,

$$E_\varphi = -\frac{J\phi_o B}{c^2(\eta - i\omega M + ik/\omega)} \tag{4}$$

Differentiating (1) with respect to time and replacing the time derivative of the vector potential,$-\partial A/\partial t = E + \nabla\varphi$, we can write,

$$\frac{dJ}{dt} = \frac{c^2}{4\pi\lambda_L^2}(E + E_\varphi) \tag{5}$$

Substituting (4) into (5) and taking the time dependence of the current as $J = J_o e^{i\omega t}$, we find,

$$J = E\left[\frac{\phi_o B}{c^2(\eta - i\omega M + ik/\omega)} - \frac{4\pi i\omega\lambda_L^2}{c^2}\right]^{-1} \tag{6}$$

so that the complex resistivity may be defined by

$$\rho^{-1} = \frac{J}{E} = \left[\frac{\phi_o B}{c^2(\eta - i\omega M + ik/\omega)} - \frac{4\pi i\omega\lambda_L^2}{c^2}\right]^{-1} \tag{7}$$

We define an effective value of the penetration depth , λ_{eff} at an effective ac frequency ω_{eff} such that

$$\lambda_{eff}^2 = \lambda_L^2(0)\left[1 + i\frac{\phi_o B}{4\pi\omega(\eta - i\omega M + ik/\omega)\lambda_L^2(0)}\right] \tag{8}$$

We define the mass term , m=$\frac{k}{\omega} - \omega M$, so that

$$\lambda_{eff} = \lambda_L(0)\left[1 + \frac{\phi_o B}{4\pi\omega\lambda_L^2(0)(m - i\eta)}\right]^{1/2} \tag{9}$$

in which the complexity is caused by the application of the magnetic field so that there is a change in the effective London penetration depth due to the magnetic field. We seperate the real and imaginary parts to define the absolute value of the London penetration depth as,

$$\lambda_{eff} = \lambda_L(0)\left[\{1 + \frac{\phi_o Bm}{4\pi\omega\lambda_L^2(0)(m^2 + \eta^2)}\}^2 + \{\frac{\phi_o B\eta}{4\pi\omega\lambda_L^2(0)(m^2 + \eta^2)}\}^2\right]^{1/4} \tag{10}$$

At large magnetic fields the quantity 1 in the first term on the right hand side is small compared with the second term so that

$$\lambda_{eff} \approx B^{1/2}(m^2 + \eta^2)^{-1/4}\left[\frac{\phi_o}{4\pi\omega}\right]^{1/2} \tag{11}$$

which predicts that at large fields the effective London penetration depth varies as the square root of the magnetic field. It has been previously found [10] that the surface resistance and reactance increase with magnetic field as a result of energy loss through fluxoids driven by superconducting currents. The change in the London penetration depth is consistent with the change in reactance upon the application of the magnetic field.

Comaparison with experimental measurements

The change in the London penetration depth, $\delta\lambda_L$, as a function of magnetic field, $B \parallel c$ and $H_{rf} \perp c$ for T=4.96 K in the single crystals of $ErNi_2B_2C$ has been deduced by rf kinetic inductance measurements by Eskildsen et al [9] who also found topological transition in the flux lattice using small-angle neutron scattering. From a plot of the logarithm of the form-factor versus the magnetic field, it is found that $\lambda \approx 500\mathring{A}$ and coherence length, $\xi = 135\mathring{A}$ at T=2.2 K. We have extracted the measured values of $\delta\lambda_L$ from their measurements and plotted them in Fig.1 as a function of $B^{1/2}$.

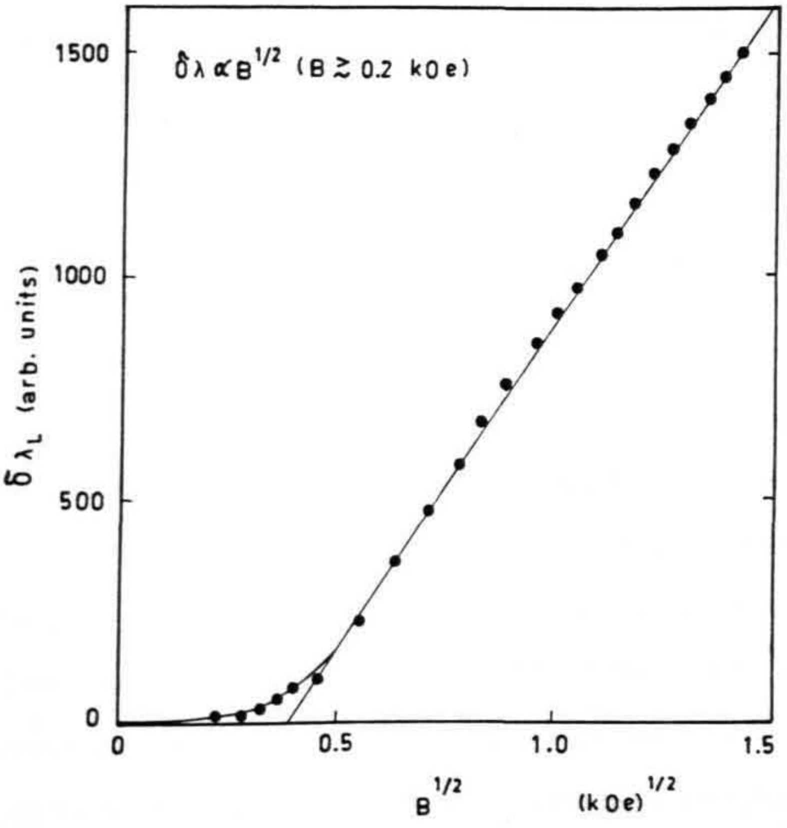

Fig.1 Change in the London penetration depth of single crystals of $ErNi_2B_2C$ as a function of square root of the magnetic field. The dots have been drawn on the basis of experimental measurements performed by Eskildsen et al [9]. The straight line showing $\delta\lambda_L \propto B^{1/2}$ is based on the theory given in the text.

It is clear that there are two seperate mechanisms. One at low magnetic fields, $B < 0.2\,kOe$, where the field dependence of $\delta\lambda_L$ is a slowly varying function of magnetic field and another at high magnetic fields, $B > 0.2\,kOe$, at which the measured values are proportional to the square root of the field. The penetration depth as well as the coherence length has also been measured [11] in YNi_2B_2C and $LuNi_2B_2C$ single crystals with $\lambda_L = 1060 \pm 30\mathring{A}$ for both the compounds and $\xi = 88 \pm 3\mathring{A}$ for Y and $82 \pm 2\mathring{A}$ for Lu compound. The properties of the Y compound are also reported by Tomy et al [12] . It is obvious that at high fields the measured values are in good agreement with that predicted by the theory of eq.(11) based on the viscous damping of vortices in superconductors. The pinning force constant is reduced to $k - \omega^2 M$ due to kinetic motion of vortices. Thus the observation of the variation of the penetration depth as a function of square root of field shows that the flux is moving in amedium of pinning forces damped by the viscous motion.

Conclusions

Usually the London penetration depth determines the distance up to which the magnetic field enters the superconductor according to exponential decay, $B = B_o e^{-x/\lambda_L}$. This penetration depth is independent of temperature and field. However, due to wave vector space, a small temperature dependence is introduced which depends on the symmetry of the gap [1]. The boson nature of vortices giving rise to quantum effects in vortices has also been reported [13]. The electromagnetic contribution to the penetration depth also depends on temperature [8]. In the present work, we have shown that the effective penetration depth varies as the square root of the magnetic field due to the viscous forces on the fluxoids.

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Chapter-IV

Magnetic field dependence of the penetration depth in superconductors

The penetration depth for a superconductor is calculated. For large viscosity it depends on the square root of the field. When the viscosity is zero, it becomes linear in magnetic field. The magnetic field dependence for a viscous medium agrees with that measured in $ErNi_2B_2C$ and that for a nonviscous medium agrees with that in $NbSe_2$ and $YBa_2Cu_3O_7$. When quantized phase of the Josephson current is considered, the penetration depth shows oscillations as a function of magnetic field. Such an oscillatory dependence is indicated in the experimental measurements of muon spin rotation of $CeRu_2$. The Josephson voltage depends linearly on the vortex velocity which becomes complex so that there is a possibility to switch from one velocity to another. Voltage measurements along the c-axis in $Bi_2Sr_2CaCu_2O_{8+y}$ appear to jump from one velocity to another just as found by theoretical calculations.

Introduction

According to the BCS theory [1] the diamagnetic current is determined by the vector potential of the electromagnetic wave, so that the superfluid density depends on the wave vectors. The summation over the wave vector can be changed to integration with the use of density of states. Therefore, the density of states plays an important role in determining the magnetic field dependence of the superfluid density. Since the diamagnetic current,

$$J = -\frac{c}{4\pi\lambda_L^2}A \tag{1}$$

can be written as

$$J = -\frac{e^2}{mc}\sum_{k,q,\sigma} c^\dagger_{k+q,\sigma}c_{k,\sigma}e^{-iq.x}A(x) \tag{2}$$

we can obtain the London penetration depth from the superfluid density,

$$\frac{1}{\lambda_L^2} = \frac{4\pi e^2}{mc^2}\sum_{k,q,\sigma} c^\dagger_{k+q,\sigma}c_{k,\sigma}e^{-iq.x} \tag{3}$$

Here J is the diamagnetic current. $\vec{A}$ is the vector potential of the electromagnetic wave and λ_L is the London penetration depth. The charge of the electron is e and m is the mass of the electron. The velocity of light is c. The creation operator for an electron of wave vector k and spin σ is $c^\dagger_{k,\sigma}$. The hermitian conjugate of which is the annihilation operator $c_{k,\sigma}$. We multiply the right hand side of the above result by density of states to replace the summation $\sum_k$ to integration $\int \cdots dk$. Won and Maki [2] have shown that the density of states for a ground state of d-wave symmetry varies with the square root of the magnetic induction,

$$\frac{N(B,0)}{N(0)} \propto \left(\frac{B}{H_{c2}}\right)^{1/2} \tag{4}$$

so that we predict, $\lambda_{L,d} \propto B^{-1/4}$. In the case of s-wave symmetry we expect that the density of states varies linearly with magnetic field. Therefore, the London penetration depth varies as the inverse square root of the magnetic field, $\lambda_{L,s} \propto B^{-1/2}$. The operator part of the integral, $c^{\dagger}_{k+q,\sigma}c_{k\sigma}$ gives rise to a factor which depends on the Fermi distribution, $(1-f_{k+q,\sigma})f_{k,\sigma}$, and hence involves magnetic field through the single-particle energy. At low temperatures, the effect of this factor on the magnetic field dependence of the London penetration depth is quite small. Actually none of the experimental measurements agree with such a dependence on magnetic field. Therefore, it is clear that some more elementary approach is needed than is described by the BCS theory with diamagnetic current and suitable density of states.

Yip and Sauls [3] have suggested that the pair breaking effect of the magnetic field reduces the current so that the penetration depth as a function of field and temperature is given by,

$$\frac{1}{\lambda(T,H)}=\frac{1}{\lambda(T)}\left[1-\frac{1}{3}\alpha\frac{H^2}{H_o^2}\right] \tag{5}$$

which diverges at $H=H_o(3/\alpha)^{1/2}$ where $H_o=\frac{3}{4}cv_c/e\lambda(T)$. Here c is the velocity of light and v_c is the critical velocity of the vortex. Apparently such a divergence has not been found in experimental measurements. In the case of anisotropic Fermi surface due to d-wave symmetry of the gap, the effective penetration depth is shown [3] to become,

$$\frac{1}{\lambda(T,H)}=\frac{1}{\lambda(T)}\left(1-\frac{2}{3}\frac{H}{H_o}\right) \tag{6}$$

which diverges at $H=3H_o/2$. This divergence is also not found experimentally. It will be quite reasonable if such a divergence occured at H_{c1} or H_{c2} which are the lower and the upper critical fields of the superconductor. Franz et al [4] have considered the effect of the gap on the penetration depth and have also found the vortices as a function of field but the value of $\lambda(T,H)$ was not evaluated analytically. Similarly, Affleck et al [5] have considered the vortex lattice structure using a generalized London free energy. Although, the s and the d wave amplitudes affected the value of the penetration depth, the magnetic field did not have a pronounced effect. It is found by Kosztin and Leggett [6] that the penetration depth of a pure *d*-wave superconductor is proportional to T^2 due to nonlocal electrodynamics. An effort has been made by Coffey and Clem [7] to understand the temperature and field dependence of the penetration depth using vortex viscosity and the earlier result of Campbell [8] which used vortex oscillations.

In this chapter, we show that under certain conditions the London penetration depth depends on the square root of the applied magnetic field which becomes linear upon expansion. The tunneling along the *c*-axis gives rise to two resolvable components of the velocity of the vortex. The phase factor gives rise to oscillations in the measurement of the London penetration depth as a function of magnetic field. We find that the theoretical predictions are in accord with the experimental measurements in $ErNi_2B_2C$, $NbSe_2$, $YBa_2Cu_3O_{7-\delta}$, $Bi_2Sr_2CaCu_2O_{8+y}$ and $CeRu_2$.

Vortex velocity

There are vortices in the type-II superconductors. These vortices oscillate and have viscous forces [9]. The mass of the ordinary fluxoid is M so that it is subject to a force $M\frac{dv}{dt}$. The forces are balanced by the current J, so that,

$$M\frac{dv}{dt} + \eta v + kx = \frac{1}{c}\,J\phi_o \tag{7}$$

where η is the viscosity and k is the force constant. The vortex velocity is dependent on time, $v = v_o e^{-i\omega t}$ which we substitute in (7) and solve to find,

$$v = \frac{J\phi_o}{c\{\eta - i\omega M + (ik/\omega)\}} \tag{8}$$

The vortex moving with velocity v in a magnetic field B produces the electric field,

$$E_\varphi = -\frac{1}{c}vB \tag{9}$$

The London penetration depth is defined by the relation,

$$\frac{dJ}{dt} = \frac{c^2}{4\pi\lambda_L^2}(E + E_\varphi) \tag{10}$$

where $E + \nabla\varphi = -\partial A/\partial t$. Substituting (8) in (9) and the resulting relation into (10) and taking $J = J_o e^{-i\omega t}$, we find

$$J = E\left[\frac{\phi_o B}{c^2\{\eta - i\omega M + (ik/\omega)\}} - \frac{4\pi i\omega\lambda_L^2}{c^2}\right]^{-1} \tag{11}$$

which can be used to define the resistivity or the conductivity, $J/E = 1/\rho$ or

$$\rho = \frac{\phi_o B}{c^2\{\eta - i\omega M + (ik/\omega)\}} - \frac{4\pi i\omega\lambda_L^2}{c^2} \tag{12}$$

Separating the real and imaginary parts we find that the complex resistivity is given by

$$\rho = \frac{\phi_o B\eta}{c^2(\eta^2 + m^2)} - i\left[\frac{m(\phi_o B/c^2)}{\eta^2 + m^2} + \frac{4\pi\omega\lambda_L^2}{c^2}\right] \tag{13}$$

where $m = \frac{k}{\omega} - \omega M$. From the beginning of the problem, we could have written $n\phi_o$ in eq.(7) where n is an integer. This means that the resistivity oscillates and the values of n give quantized resistivity. Some time ago the flux was quantized [10] in the units of $(n + \frac{1}{2})\phi_o$ so that the minimum flux becomes $\phi_o/2$. However, recently we have found [11] that in paramagnetic Meissner effect the minimum flux observed is $\phi_o/4$. Therefore, in a device the resistivity may be quantized in units of $(p/q)\phi_o$ from the first term in (13) where p and q are any integers. The last term in (13) depends on the penetration depth as if the heat loss which is $J^2 Im\rho$ is caused by the penetration of field. Here $Im\rho$ is the imaginary part of ρ. The term previous to the one dependent on λ_L^2 actually depends on m so that part of the heat loss is caused by the oscillations of the vortex but the term dependent on mass causes retardation and opposes this effect.

We define an effective London penetration depth

$$\lambda_{eff}^2 = \lambda^2(o)\left[1 + \frac{i\phi_o B}{4\pi\omega\{\eta - i\omega M + (ik/\omega)\}\lambda_L^2(o)}\right] \tag{14}$$

which can be written as

$$\lambda_{eff} = \lambda_L(o)\left[1 + \frac{\phi_o B}{4\pi\omega\lambda_L^2(o)(m - i\eta)}\right]^{1/2} \tag{15}$$

where the complexity is caused by the application of the magnetic field. We separate the real and imaginary parts to define the absolute value of the London penetration depth as

$$\lambda_{eff} = \lambda_L(o)\left[\left\{1 + \frac{\phi_o B m}{4\pi\omega\lambda_L^2(o)(m^2 + \eta^2)}\right\}^2 + \left\{\frac{\phi_o B \eta}{4\pi\omega\lambda_L^2(o)(m^2 + \eta^2)}\right\}^2\right]^{1/4} \tag{16}$$

This expression is of general nature. It can give rise to the square root of the magnetic field. When binomially expanded, it gives a linear function of magnetic field. The full phase factor introduced in ϕ_o due to Josephson tunneling can bend the $\lambda(H)$ curve from the linear behaviour and introduction of quantized flux, $n\phi_o$, instead of ϕ_o, leads to quantized resistivity. We consider several special cases.

Case I: At large fields

$$\lambda_{eff} = B^{1/2}(\phi_o/4\pi\omega)^{1/2}\left[\left(\frac{k}{\omega} - \omega M\right)^2 + \eta^2\right]^{-1/4} \tag{17}$$

and for large η,

$$\lambda_{eff} = B^{1/2}(\phi_o/4\pi\omega)^{1/2}\eta^{-1/2} \tag{18}$$

which shows that the effective penetration depth depends on the square root of the magnetic field.

CaseII. For small fields, we obtain the binomial expansion,

$$\lambda_{eff} = \lambda(o)\left[1 + \frac{\phi_o B}{8\pi\omega\lambda_L^2(m - i\eta)}\right] \tag{19}$$

which for small viscosity, $\eta = 0$, becomes,

$$\lambda_{eff} = \lambda(o)\left[1 + \frac{\phi_o B}{8\pi\omega\lambda_L^2 m}\right] \tag{20}$$

For zero vortex mass, $M = 0$, $m = k/\omega$,

$$\lambda_{eff} = \lambda(o)\left[1 + \frac{\phi_o B_{c2}}{8\pi\lambda_L^2 k}\left(\frac{B}{B_{c2}}\right)\right] \tag{21}$$

Introducing dimensionless variable $h = B/B_{c2}$ and

$$\beta = \frac{\phi_o B_{c2}}{8\pi k\lambda_L^2(o)} \tag{22}$$

we find that the penetration depth depends linearly on the magnetic field

$$\lambda_{eff} = \lambda(o)[1 + \beta h] \tag{23}$$

Case III. In the superconductors with weak links, the critical current is replaced by the Josephson current with quantized phase so that on the right hand side of eq.(1), we can add a phase factor. For small magnetic fields, we expand the cosine term, retaining only two terms,

$$\frac{J\phi_o}{c}\cos\left(\frac{2\pi}{\phi_o}\phi_x\right) \approx \frac{J\phi_o}{c}\left[1 - \frac{2\pi^2\phi_x^2}{\phi_o^2}\right] \tag{24}$$

so that the vortex velocity becomes,

$$v = \frac{J\phi_o - 2J\pi^2 A^2 B^2 \phi_o^{-1}}{c[\eta - i\omega M + (ik/\omega)]} \tag{25}$$

where flux is quantized within the area A such that $BA = \phi_x$. This velocity produces the electric field of

$$E_\varphi = \frac{-J\phi_o B + J(2\pi^2 A^2\phi_o^{-1})B^3}{c^2[\eta - i\omega M + (ik/\omega)]} \tag{26}$$

from which the effective penetration depth is found to become,

$$-\frac{4\pi i\omega}{c^2}\lambda_{eff}^2 = -\frac{4\pi i\omega}{c^2}\lambda_L^2 + \frac{\phi_o B}{c^2[\eta - i\omega M + (ik/\omega)]} - \frac{(2\pi^2 A^2\phi_o^{-1})B^3}{c^2[\eta - i\omega M + (ik/\omega)]} \tag{27}$$

Upon comparing with (14) we find that the field is

$$B_{eff} = B\left[1 - \frac{2\pi^2 A^2 B^2}{\phi_o^2}\right] \tag{28}$$

so that in the approximation which leads to the square root of the field as in (18), the penetration depth varies as

$$\lambda_{eff} \propto B^{1/2}[1 - aB^2]^{1/2} \tag{29}$$

where $a = 2\pi^2 A^2/\phi_o^2$ and in the approximation which gives penetration depth linear in field,

$$\lambda_{eff} \propto B[1 - aB^2] \tag{30}$$

in which the bending in $\lambda_{eff}(B)$ varies as B^3. Retaining the full cosine term in the resistivity, it is seen that

$$\rho = -\frac{4\pi i\omega\lambda_L^2}{c^2} + \frac{\phi_o B}{c^2\eta}\cos[2\pi\phi_x/\phi_o] \tag{31}$$

which oscillates as a function of magnetic field. We observe that ρ is proportional to the square of the penetration depth. Thus oscillations occur in the square of the penetration depth as a function of magnetic field. In the eq.(31), ϕ_o may be replaced by $n\phi_o$ where n is an integer. Then it is seen that resistivity is quantized,

$$\rho \approx nB \tag{32}$$

in units of $\phi_o/c^2\eta$ leaving out the phase factor as equal to unity and the first term which does not depend on the field. Similarly, the square of the effective London penetration depth, $\lambda_{eff}^2 \propto nB$, gets quantized.

Case IV. The voltage as a function of field can be determined by our theory. The electric field generated by the motion of fluxoids in the magnetic field is determined by

$$E = -\frac{J\phi_o B(\eta - im)}{c^2(\eta^2 + m^2)} \tag{33}$$

The voltage is applied across a distance r so that, $V = Er$, is the voltage. Differentiating this voltage with respect to the magnetic field B, we find the effective vortex velcoity,

$v = \partial V/\partial B$ as,

$$v = \frac{-J\phi_o r(\eta - im)}{c^2(\eta^2 + m^2)} = -v_1 + iv_2 \tag{34}$$

with

$$v_1 = \frac{J\phi_o \eta r}{c^2(\eta^2 + m^2)} \tag{35}$$

$$v_2 = \frac{J\phi_o m r}{c^2(\eta^2 + m^2)} \tag{36}$$

The real part v_1 is nonzero even if $m = 0$. This is the velocity which can be greater than the velocity of light in the medium for

$$\frac{J\phi_o \eta r}{\eta^2 + m^2} > c^3 \tag{37}$$

For small m, $(J\phi_o r/\eta)^{1/3} > c$ in a medium is allowed and v_2 may also be larger than c. The ratio of the two velcoities $v_1/v_2 = \eta/m$ indicates that one of the two velocities may be greater than c depending of the ratio η/m. The system is thus in a dynamic equilibrium and can switch from one velocity to another.

Comparison with experimental results

(a) Eskildsen et al [12] have obtained the change in the penetration depth in single crystal of $ErNi_2B_2C$ as a function of magnetic field by r.f. kinetic inductance measurements. We have found that except at small fields, the measured values are in accord with the predicted dependence of the penetration depth on the square root of magnetic field as shown by the expression (18), $\lambda_L \propto B^{1/2}$, for large viscosity. In Fig.1 we show the measured values of the penetration depth as a function of the square root of the magnetic field which agree with the theory.

(b) In the case of small viscosity, the equations (21) and (23) predict that the penetration depth is linearly proportional to the magnetic field. Sonier et al [13] have

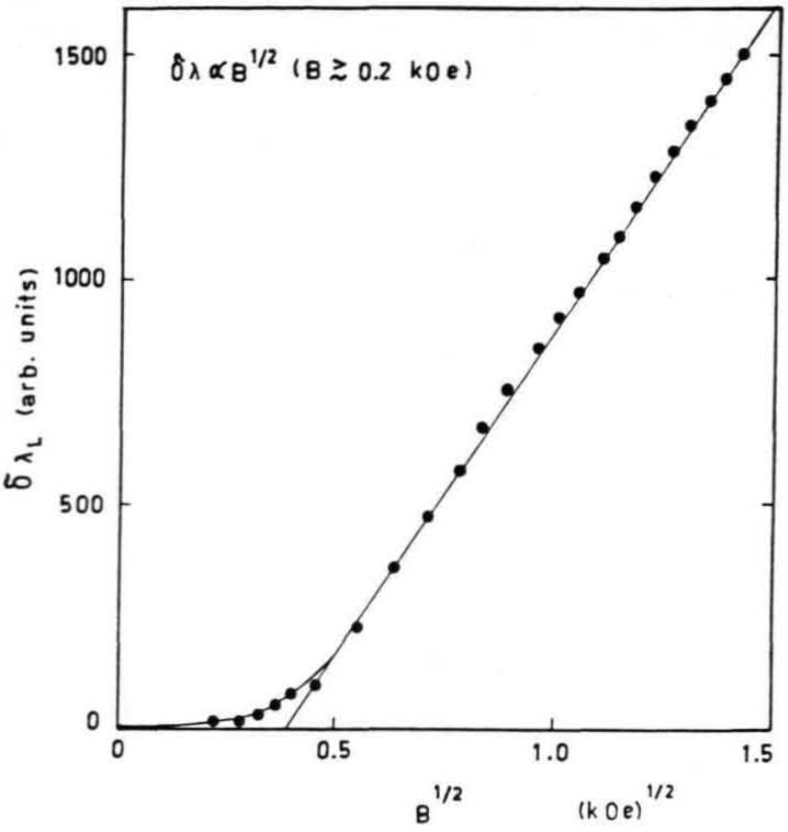

Fig.1 The measured changes in the penetration depth in single crystals of $ErNi_2B_2C$ shown by dots, taken from the work of Eskildsen et al [12], are plotted against the square root of the magnetic field. It is found that except at small fields, the change in penetration depth is proportional to the square root of the magnetic field.

measured the magnetic field dependence of the penetration depth $\lambda_{ab}(H)$ in the vortex state of $NbSe_2$ at 0.6 T_c and also at 0.33 T_c. In both the cases the penetration depth is seen to linearly increase with increasing magnetic field. In particular the data agrees with eq. (23) for

$\lambda_{ab}(o) = 1323\,\mathring{A}$, $\beta = 1.61$ at $T = 0.33T_c$ and

$\lambda_{ab}(o) = 1436\,\mathring{A}$, $\beta = 1.56$ at $T = 0.60T_c$.

The data for $T = 0.33T_c$ are displayed in Fig.2 showing the amount of agreement between the theory and the experimental measurements.

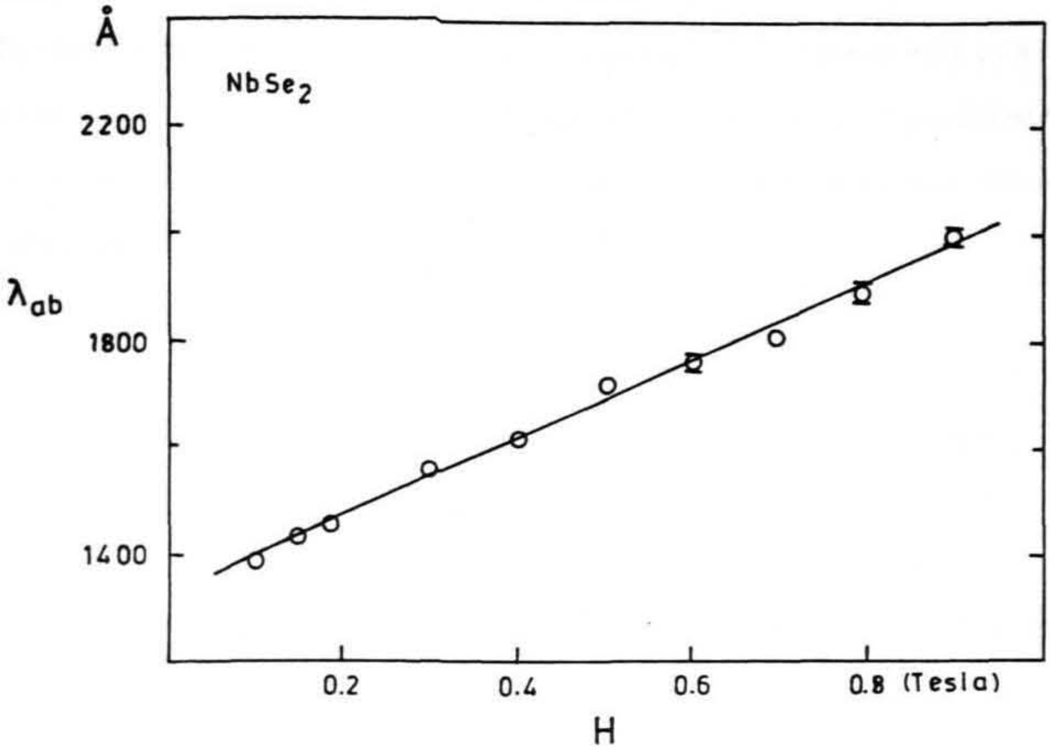

Fig.2 The measured values of the penetration depth in $NbSe_2$ are seen to be linearly proportional to the magnetic field for $T = 0.33T_c$. The data are taken from Sonier et al [13] and the calculation is given by eq.(23).

Sonier et al [14] have also measured the $\lambda_{ab}(H)$ in $YBa_2Cu_3O_{6.6}$ in both the twinned as well as the detwinned samples. For twinned samples, $H_{c2} = 70T$, $\lambda_{ab}(0,0) = 1586\,\mathring{A}$ and $\beta = 6.6$ and for the detwinned samples $\lambda_{ab}(0,0) = 1699\,\mathring{A}$ and $\beta = 5.0$. In both the cases eq.(23) is well obeyed.

(c) The magnetic field penetration depth λ_L in $CeRu_2$ has been obtained using transverse field μSR by Yamashita et al [15]. At 2K for a field of 0.9846 T the penetration depth is found to be 1983±15 Å. As the field is increased to 1.9621 T the penetration depth increases to 2101±11 Å but upon further increase of field to 2.9312 T the penetration depth reduces but increases to 2937± 24 Å upon further increase of field to 3.9003 T. This type of oscillatory behaviour is qualitatively in accord with that predicted by (20) and is caused by the phase factor dependent term in the tunneling current. Considering the phase factor (20) becomes,

$$\lambda_{eff} = \lambda(o)\left[1 + \frac{\phi_o B}{8\pi\omega\lambda_L^2 m}\cos\frac{2\pi\phi_x}{\phi_o}\right] \tag{38}$$

Between two fields B_1 and B_2 the cosine term shows one half oscillation when

$$\left(\frac{2\pi\phi_{x1}}{\phi_o} - \frac{\pi}{2}\right) - \left(\frac{2\pi\phi_{x2}}{\phi_o} - \frac{\pi}{2}\right) = \pi \tag{39}$$

Since the field is quantized in an area A, we write $\phi_{x1} = AB_1$ and $\phi_{x2} = AB_2$ so that one half oscilaltion is obtained when

$$B_1 - B_2 = \frac{\phi_o}{\pi A} \tag{40}$$

In Fig.3 we have shown the experimental data points [15] of penetration depth as a function of field in $CeRu_2$ along with a qualitative curve drawn as a guide to the eyes. It is found that the data are in qualitative agreement with the interpretation given above. The period of oscillations in Fig.3 is about 2×10^4 Gauss. Substituting this value for $B_1 - B_2$ and the value of ϕ_o we find that $A^{1/2} \simeq (10/\pi)^{1/2}10^{-6}cm$ which is quite reasonable for the size of the current loops.

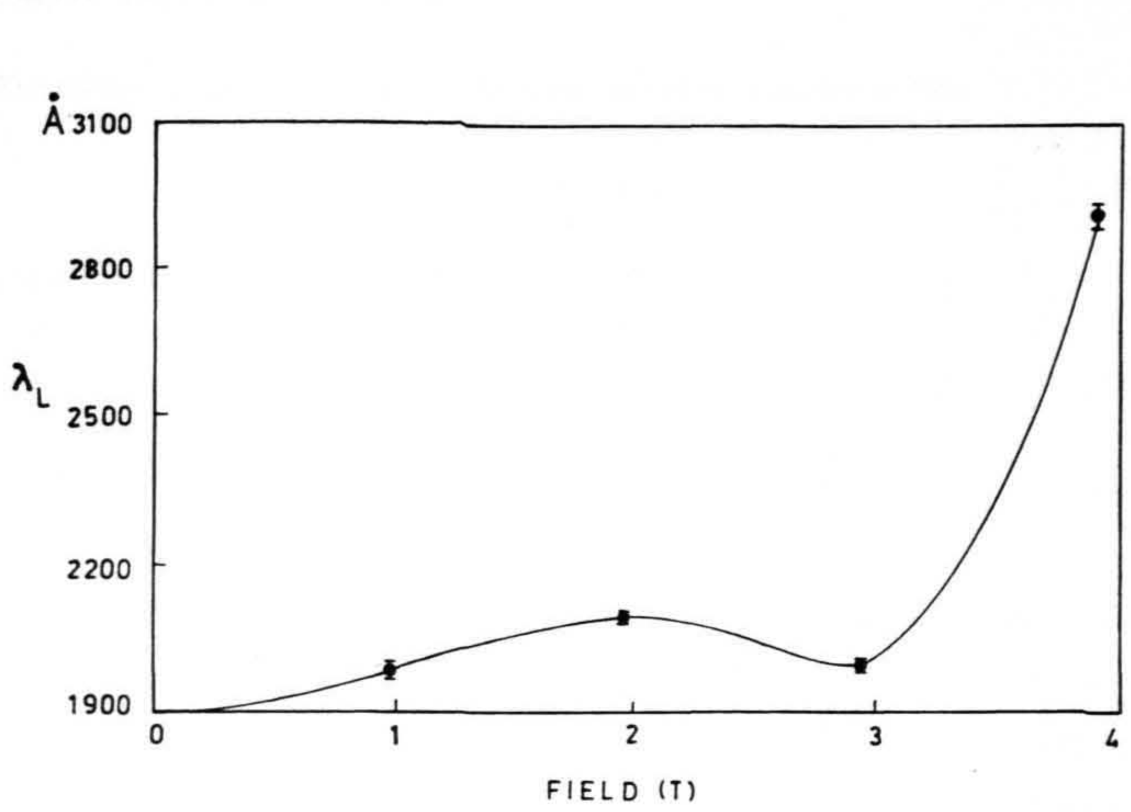

Fig.3 Oscillation of period of about 2T is found in CeRu$_2$. The experimental data is taken from Yamashita et al [15] and the theory is given by eq. (38).

(d) For Josephson junctions the voltage frequency relation is $2eV = h\nu$ from which the voltage is found to be

$$V = \frac{hc}{2e}\frac{\nu}{c} = \phi_o\frac{\nu}{c} \tag{41}$$

where V is the applied voltage and ν is the microwave frequency. The voltage can be written in terms of the unit flux, $\phi_o = hc/2e$. For a stack of N intrinsic Josephson junctions

$$V = N\phi_o\frac{\nu}{c} \ . \tag{42}$$

The voltage is being applied across the thickness t of the junction so that

$$2eV = 2eB\bar{v}t \tag{43}$$

where $\bar{v}$ is the standard Swihart velocity [16-18]

$$V = B\bar{v}t \tag{44}$$

so that for N junctions

$$V = N\bar{v}Bt\ . \tag{45}$$

The plot of voltage as a function of field in a stack of N Josephson junctions is thus predicted to be linear with only one slope. It has been shown in (34) that the velocity becomes complex so that the system can jump from one state with velocity v_1 to another state of velocity v_2. The c-axis in $Bi_2Sr_2CaCu_2O_{8+y}$ is analogous to a stack of N Josephson junctions. The tunneling voltage along this direction as a function of field measured by Hechtfischer et al [19] shows two slopes as expected for a system which switches from v_1 to v_2 at a field of 17 kOe. Therefore, the measurements are in accord with the theory. It may be noted that due to the flux lattice and its melting [20], the ordinary expression for the London penetration depth requires Stephen-Bardeen corrections owing to the viscosity and oscillations in the vortex lattice. Portis et al [21] have also pointed out the importance of the term containing mass of the vortex to determine the surface resistivity which measures the penetration depth.

Conclusions

The penetration depth in type-II superconductors is determined by viscous oscillatory forces. For large viscosity the change in penetration depth as a function of magnetic field depends on the square root of the magnetic field. For small viscosity, the penetration depth depends linearly on the magnetic field. For intermediate cases due to quantized phase factor we expect oscillations in the penetration depth as a function of magnetic field. The vortex velocity becomes complex when viscous forces are comparable with those of oscillatory motion of the vortex. In such a case a transition from one vortex velocity to another vortex velocity is predicted. It may be noted that the current depends [22] on the symmetry of the gap of the superconductor. Therefore the square of the penetration depth depends on the symmetry of the superconducting state which requires detailed investigtation. All of the theoretical results are found to be in agreement with various measurements in $ErNi_2B_2C$, $NbSe_2$ and $YBa_2Cu_3O_7$, $CeRu_2$ and c-axis voltage in $Bi_2Sr_2CaCu_2O_8$.

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Chapter-V

Soft vortices in type-II superconductors: YNi_2B_2C and $ErNi_2B_2C$

We show that the apex angle of the rhombic cell of YNi_2B_2C decreases as a function of increasing magnetic field upto a certain constant field, $B_o \simeq 1130 \pm 70$ Oe. This dependence is consistent with the mean field theory of soft vortices. In $ErNi_2B_2C$, the magnetic penetration depth, also exhibits a soft vortex type behaviour below $B_o \simeq 250 \pm 10$ Gauss. The penetration depth is found to have a component which varies as $B^{1/2}[1 - (B/B_o)]^{-1/4}$ as predicted by the mean field theory and shows divergence at B_o. In both the samples, the theoretical predictions are in accord with the experimental measurements. This is the first time that London penetration depth associated with a soft vortex has been reported.

Introduction

Recently, it has been found that yttrium and rare-earth borocarbides upon cooling become conducting and upon further cooling become superconducting. There is a microscopic coexistence of antiferromagnetism and superconductivity in some of these compounds. In YNi_2B_2C, upon the application of a magnetic field, the hexagonal flux lattice changes into a square configuration above a certain field [1]. The apex angle, β, of the orthorhombic cell as a function of magnetic field decreases upon increasing field upto a certain field above which it becomes a constant. There is a vortex lattice reorientation transition from a state with the diagonal of the rhombic unit cell along [110] at low fields to [100] direction at high fields above the characteristic field. In $ErNi_2B_2C$, the high field square lattice transforms into a hexagonal lattice below a characteristic field [2]. The high field square lattice is aligned with [110] direction of the tetragonal crystal, while the low field hexagonal lattice is aligned with [100]. Within a certain range of fields both the square and hexagonal lattices are seen to coexist. In both the crystals YNi_2B_2C and $ErNi_2B_2C$ the square configuration is stable at high field while hexagonal configuration is found at low fields. The area of the square lattice is equal to that of the hexagonal lattice in the coexistence region of the magnetic field for a given field in separate domains due to the flux quantization. These observations are analogous to that of a soft mode accompanied with a lattice distortion. The square lattice of higher symmetry is found at higher fields while lower symmetry occurs at lower fields.

Kogan et al. [3] have calculated the nonlocal corrections to the London model of a superconductor. In the case of $LuNi_2B_2C$ they report that the apex angle β of the rhombic unit cell shows two values for which the free-energy is a minimum. The apex angle as a function of magnetic field appears to increase with increasing magnetic field upto a certain field. Above this field the apex angle becomes independent of the field.

Kogan et al. [4] also report that theoretical models of nonlocal corrections to the London equations reproduce the symmetries of vortex lattices.

In this chapter we show that the apex angle of the orthorhombic cell exhibits a mean field exponent, $\beta = \beta_o[1-(B/B_o)]^{1/2}$ with $\beta_o = 8^o$ and $B_o = 1130 \pm 70$ Oe as a function of field, in YNi_2B_2C. Since the magnetic penetration depth varies as the inverse square root of the frequency, we predicted that change in penetration depth varies as $\delta\lambda_L \propto B^{1/2}[1-(B/B_o)]^{-1/4}$. For small fields, $B < 250$ Oe, we compare the experimental measurements of the change in London penetration depth in $ErNi_2B_2C$ with the theory using mean field exponents. The measured values are in reasonable agreement with the theory. For large fields, 2250> B >250 Oe, the penetration depth varies as the square root of the magnetic field in agreement with the theory which uses the viscous vortex oscillations.

Soft vortices

In the case of soft modes the higher temperature phase has higher symmetry and the lower temperature phase has lower symmetry. The soft lattice modes are often detected in the Raman spectra of solids. In the case of vortex lattices, we expect that the displacement of vortices should vary as

$$\delta x = (\delta x)_o[1-(T/T_c)]^{1/2} \tag{1}$$

where mean field value for the critical exponent is chosen. In some of the studies, instead of the displacement an angle is measured as a function of temperature which also varies with temperature according to mean field exponent [5]. Similarly, in magnetic systems we expect soft modes so that the displacement of magnetic atoms or the angles vary with field,

$$\delta x = [\delta x]_o[1-(B/B_o)]^{1/2} \tag{2}$$

where B_o is the critical induction. The frequency of the soft mode vanishes at the critical temperature and at the critical field,

$$\omega = \omega(0)[1 - (T/T_c)]^{1/2} \tag{3}$$

and

$$\omega = \omega_o[1 - (B/B_o)]^{1/2} \tag{4}$$

The crystallographic angles are expected to vary as the distance between atoms as given above so that in orthorhombic crystals,

$$\beta = \beta_o[1 - (B/B_o)]^{1/2} \tag{5}$$

Stephen and Bardeen [6,7] have shown that the vortices are subject to a viscous force which contributes to the current. Coffey and Clem [8] have found the complex penetration depth in terms of the field-dependent penetration depth, the normal-fluid skin depth and the effective skin depth. We have solved [9] the equation of motion of a vortex properly to define an effective penetration depth in a type-II superconductor. In addition to the viscosity term in the force, we use the harmonic oscillations in the vortex and also include the mass of the vortex to determine the current. The equation is then

$$M\frac{dv}{dt} + \eta v + kx = \frac{1}{c}J\phi_o \tag{6}$$

where M is the mass of the vortex, dv/dt is the acceleration, η is the coefficient of viscosity of the vortex of velocity v, force constant k and the critical current J [10]. It was found that at large magnetic fields, the effective London penetration depth varies as,

$$\lambda_{eff} \approx B^{1/2}\left[\left\{\left(\frac{k}{\omega}\right) - \omega M\right\}^2 + \eta^2\right]^{-1/4}[\phi_o/(4\pi\omega)]^{1/2} \tag{7}$$

The relationship between the effective magnetic penetration depth and the London penetration depth is also given in [9]. It may be noted that the original London penetration depth is independent of field. However, we have found that the penetration depth changes when magnetic field is applied to a vortex. The relation (7) gives the field dependent penetration depth. For large viscosity, $\eta >> (k/\omega) - \omega M$, we retain only one term in the large bracket and substitute (4) in (7) and leave out the remaining factors in a constant consistant with Stephen-Bardeen theory [6,7] so that the effective London penetration depth becomes.

$$\lambda_{eff} = k_1 \frac{B^{1/2}}{\omega_o^{1/2}[1 - (B/B_o)]^{1/4}} \tag{8}$$

where the constant k_1 can be determined by comparing (8) with (7). If peaks are absent in the resistivity the mass term is small. However, the largest contribution of soft vortices to the London penetration depth varies with field as given by (8) and the structural parameters vary as given by (2) and (5). The result (8) ignores the mass of the vortex as small so that the first term of (6) which gives the force depending on the accelaration is neglected.

Comparison with experimental measurements

We report two experimental measurements which agree with the ideas of soft vortices which are linked with two minima in the free energy when one parameter is varied. In the present case the parameter is the magnetic field. This means that when field is varied, the system changes from one minimum value to another. The higher field minimum has higher symmetry and the lower field minimum has a lower symmetry., Therefore, the two states have a critical field, B_o. Since both phases are superconducting, $B_o < B_{c2}$.

The experimental values of the apex angle, β, of the rhombic unit cell of YNi_2B_2C are taken from [1] and compared with expression (5). It is found that the data fit very well with

$$\beta = 8[1 - (B/B_o)]^{1/2} + 52 \qquad (deg) \tag{9}$$

with $B_o = 1130 \pm 70$ Oe as a shown in Fig.1.

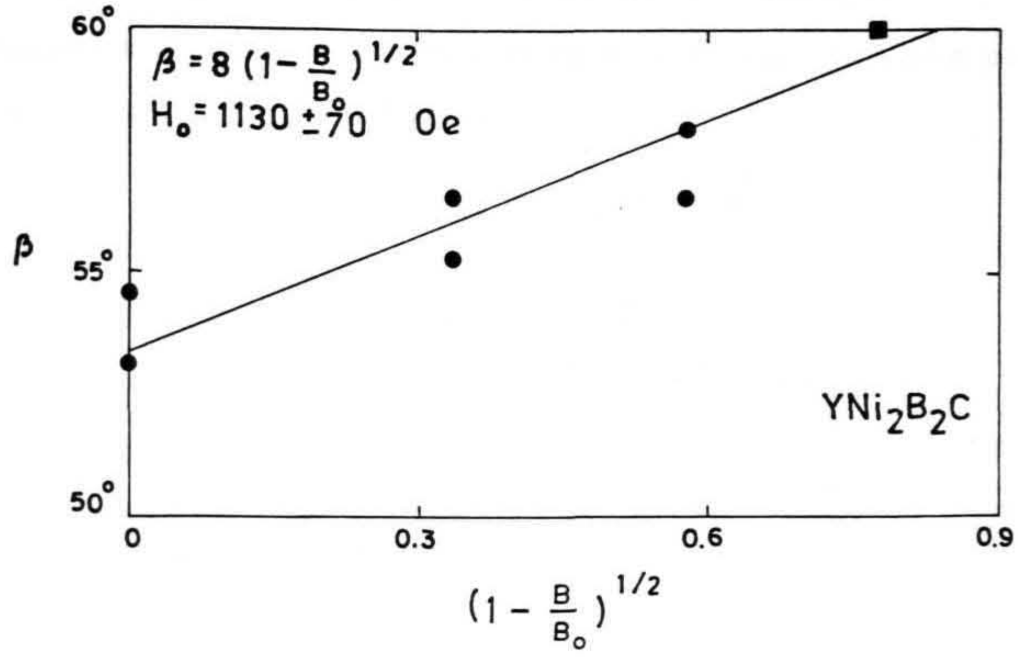

Fig.1 The apex angle β of the rhombic unit cell as a function of $[1-(B/B_o)]^{1/2}$ showing linear dependence. The experimental points are taken fromt he small angle neutron scattering measurements performed on YNi_2B_2C given in [1]. The theoretical interpretation in terms of a critical field and soft vortex is as given in the text in the present work.

We therefore conclude that in YNi_2B_2C there is a soft vortex with critical field of B_o. The angle β has a large value at $B = 0$. There is a component in this angle which reduces with increasing field and becomes zero at $B = B_o$.

In our earlier work [9] the penetration depth as a function of magnetic field in $ErNi_2B_2C$ was explained only for magnetic fields larger than ~250 Gauss. For small fields, the penetration depth increases slowly with increasing magnetic field but the measured values did not agree with the predicted $B^{1/2}$ or B^2 dependence. We are now able to explain the change in the penetration depth as a function of magnetic field for small fields, $B < 250$ G. These changes are now assigned to soft vortex modes. According to the expression (8) for magnetic fields smaller than ~250 G, we calculate the change in London penetration depth as a function of magnetic field from

$$\delta\lambda_L = C_1 B^{1/2}[1 - (B/B_o)]^{-1/4} \tag{10}$$

for $C_1 = 4.73$ and $B_o = 250 \pm 10$ G as shown in Fig.2.

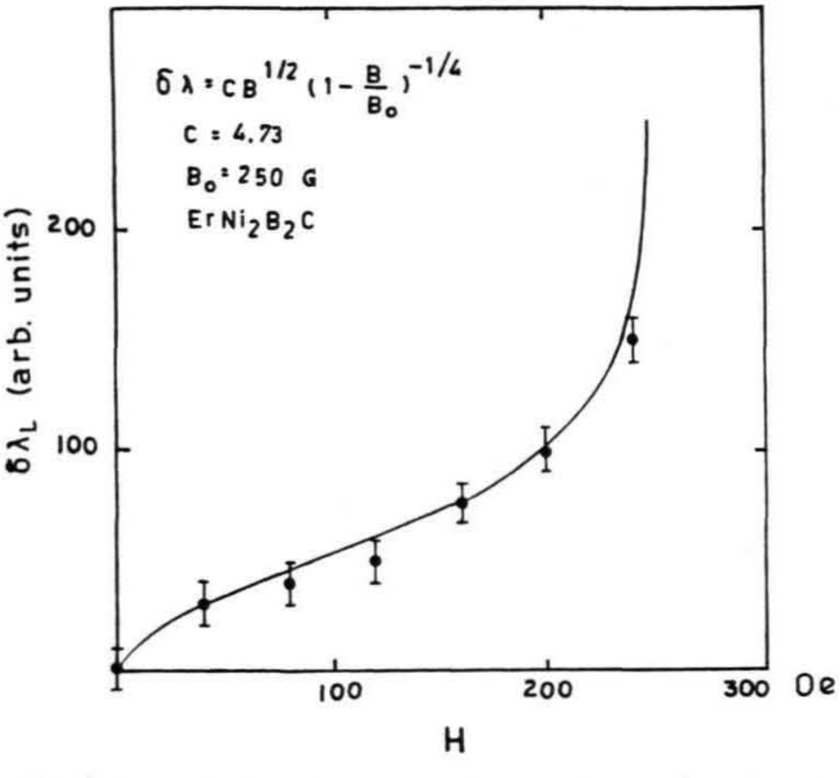

Fig.2 The change in London penetration depth as a function of magnetic field calculated for a soft vortex with critical field B_o = 250 G. There is a divergence in the calculated value at 250 G. The experimental point deduced from the measurements on $ErNi_2B_2C$ given in [1] are also shown.

The experimental values deduced from the work of Eskildsen et al. [2] for $ErNi_2B_2C$ are also shown. It is seen that the soft vortex interpretation works well and the calculated values are in agreement with the experimental data.

Conclusions

The free energy as a function of field has two minima separated by a barrier. When the applied magnetic field energy, $B^2/8\pi$, is equal to that of the barrier, there is a transition from one minimum value to another accompanied by a soft vortex. In the apex angle in YNi_2B_2C, such a soft vortex gives rise to a critical behaviour which we have described by the mean field theory. It is found that in YNi_2B_2C the apex angle varies as $[1-(B/B_o)]^{1/2}$ with $B_o \simeq 1130 \pm 70$ Oe which is in agreement with the value, $B \sim 112$ mT found in [1]. The theoretical predictions are in accord with the experimental measurements. The London penetration depth is found to vary with the magnetic field. A critical field exists such that the London penetration depth diverges at this field, $\lambda_L \propto B^{1/2}[1-(B/B_o)]^{-1/4}$. In $ErNi_2B_2C$ the measurements of penetration depth are in good agreement with the theory of soft vortices. For $B > B_o$, the vortex lattice has the symmetry of a square lattice while for $B < B_o$ the structure is hexagonal.

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Chapter-VI

Soft vortex in $TmNi_2B_2C$ superconductor

The structural distortions, magnetism and superconductivity coexist in single crystals of $TmNi_2B_2C$. It is antiferromagnetic below $T_N \simeq 1.5$ K and superconducting below $T_c \simeq 10.6$ K. We present here a neutron diffraction study which shows that the apex angle β never reaches the value of 90^0 belonging to the square lattice because of the onset of a phenomenon precursor to antiferromagnetism. At some field such as 3 K Oe upon cooling below 4 K, instead of increasing the apex angle starts reducing. The angle tends to zero at the Neel temperature and exhibits a mean field exponent of 1/2. Similarly, upon warming above 4 K, the angle becomes soft at the superconducting transition temperature, T_c. The upper critical field is highly anisotropic at low temperatures and the anisotropy indicates that the gap may exhibit a node. At elevated temperature 27 K, the upper critical field becomes isotropic as if the gap belongs to s-wave. Therefore the gap appears to have complex symmetry. The change in London penetration depth as a function of temperature has been deduced from the intensity of the neutron diffraction. This measurement also shows node like dependence at low temperatures and s-wave at high temperatures.

Introduction

The intermetallic compounds of formula RNi_2B_2C are tetragonal and are found to be superconducting with transition temperatures less than ~17 K. The values of the upper critical field are upto 10 T. The values of the transition temperature are reduced upon increasing the magnetic moment of R atoms. Thus largest T_c is found for $R = Lu$ and it reduces upon going from f^{14} to f^8 electron configuration with $R = Lu, Y, Tm, Er, Ho, Dy$ and Tb. The decrease in T_c is in the same order as the number of f electrons. Y is not a rare earth but it substitutes for a rare earth and the T_c for the Y compound is very near the value for Lu. Thus the T_c is largest for Lu compound and smallest for Tb. This decrease in T_c is interpreted in terms of electron scattering by the magnetic moments and it is consistent with Abrikosov-Gorkov theory of reduction of transition temperature by the scattering time, $\tau_{AG}, k_BT_c = k_BT_{co} - (\hbar/\tau_{AG})$. The same result is found [1] by ploting T_c with de Gennes parameter which is a measure of angular momentum, $(g-1)^2J(J+1)$. The decrease in T_c is given by,

$$\frac{\delta T_c}{T_c} \simeq nI^2N(0)(g-1)^2J(J+1)$$

where n is the number of magnetic moments, I is the exchange constant and $N(0)$ the density of states. However, the compounds of Er, Ho, Dy and Tb are ferromagnetic while that of Tm is antiferromagnetic. In this way the Tm compound has special properties. It has a very low Neel temperature, $T_N = 1.5$ K so that there is a large temperature range between T_N and the $T_c \simeq 10.6$ K in which superconducting properties can be measured. Another important property of these superconductors is the structural distortion of the flux lattice. For example, the apex angle β in YNi_2B_2C has a component [2] which varies with the application of a magnetic field. Such a component is zero at a characteristic field of 112 mT. In other words it has a soft vortex.

In this chapter, we report that $TmNi_2B_2C$ exhibits a soft vortex the value of which is zero at the superconducting transition temperature T_c as well as at the Neel temperature, T_N. As the system is cooled from T_c, it exhibits a phenomenon precursor to antiferromagnetism. Instead of the apex angle approaching 90^0 appropriate to a square lattice, it starts reducing, going towards zero at T_N. The intensity of the neutron scattering is found to be proportional to the inverse fourth power of the magnetic penetration depth. We have found that the penetration depth measured by this method is characteristic of a node in the gap of the superconductor at low temperatures where as at high temperatures $4 < T < 10.6$, it exhibits s-wave character.

Theory

The frequency of a soft mode goes to zero at a particular temperature. Therefore we expect that the displacement or the angle may also vanish at a particular temperature. In case two transitions occur, the soft vortex angle may vanish at both the transition temperatures. In the case of $TmNi_2B_2C$ the soft vortex angle may become zero at the Neel temperature as well as at the superconducting transition temperature. So the angle β is predicted to vary as,

$$\beta = \beta_1 + \beta_2 \tag{1}$$

$$\beta_1 = \beta_{10}(\frac{T_N}{T} - 1)^{\frac{1}{2}}$$

and

$$\beta_2 = \beta_{20}(1 - \frac{T}{T_c})^{\frac{1}{2}}$$

where β_{10} and β_{20} are constants, T_N is the Neel temperature and T_c is the superconducting transition temperature.

The scattering amplitude for the neutron to scatter from k_0 to $k_0 + q$ is given by,

$$a = \frac{M}{2\pi\hbar^2} \int \mu_n h(r) e^{iq.r} dr \tag{2}$$

where M is the mass of the neutron, μ_n the magnetic moment, $h(r)$ the field of the vortex lines and q the wave vector of the neutron. The integral in above is

$$\int h(r) e^{ik.r} dr = \frac{BV}{1 + \lambda^2 k^2} = \frac{n_L \phi_0 V}{1 + \lambda_L^2 k^2} \tag{3}$$

where B is the magnetic field, V the volume and λ_L the London penetration depth. The flux lines per unit area is n_L=NL/(area of the unit flux lattice). Under circumstances relevant to the experiments $1 << \lambda^2 k^2$ so that we can ignore 1 in the denominator of the scattering amplitude. Therefore,

$$a \simeq \frac{n_L V}{k^2 \lambda_L^2} \tag{4}$$

or the intensity of the neutron scattering is given by

$$I \simeq |a|^2 \simeq (\frac{n_L V}{k^2})^2 \frac{1}{\lambda_L^4} \tag{5}$$

which varies inversely as the fourth power of the London penetration depth. It has been suggested that due to the coherence the scattering amplitude may be multiplied by an exponential factor so that (4) becomes,

$$a \simeq \frac{n_L V}{k^2 \lambda_L^2} e^{-\xi^2 k_0^2 / 2(m_1 m_3)^{1/2}} \tag{6}$$

where ξ is the coherence length and m_1 and m_2 are the factors which meaure the anisotropy of the mass of a vortex. For example, the isotropic induction field B may be replaced by $[(m_1 B)^2 + (m_3 B)^2]^{1/2}$ in the x-z plane as found by Kogan [3]. The logarithm of (6) as a function of B leads to a direct meaurement of ξ or the upper critical field.

Experimental measurements

The neutron diffraction measurements on the flux-line lattice in $TmNi_2B_2C$ as a function of applied magnetic field and as a function of temperature have been performed by Paul et al [4]. One of the measurements is given in Fig.1 at $H = 3K$ Oe. For $10 > T > 4$ K, we interpret that the apex angle β becomes soft, therefore β is plotted as a function of $[1 - (T/T_c)]^{1/2}$ in Fig.2 where the exponent of 1/2 is assumed on the basis of mean field theory [5]. It is clearly seen that the temperature dependence of the apex angle fits with the soft mode interpretation very well. In order to see if the angle becomes soft also at T_N we have plotted the angle β in the range $1.5 < T < 4$ K as a function of $(\frac{T}{T_n} - 1)^{1/2}$ in Fig.3. It is found that the measured values fit very well and the angle does become soft at T_N. Neutron diffraction as a function of temperature shown in Fig.4. Since the intensity vanishes near the transition temperature, we interpret that the diffraction is indeed coming from superconducting properties of the lattice, in this case the flux line lattice. We find the inverse fourth power of the intensity which is proportinal to the London penetration depth in arbitrary units and plot it as a function of temperature. The low temperature, $T < 4$ K part is indeed linear as we expect [6] for a node in the gap, $\Delta = \Delta_0 \cos 2\phi$. The high temperature part is characteristic of s-wave symmetry, $\Delta = \Delta_0$. In Fig.5 we show the measured as well as the calculated values of the London penetration depth as a function of temperature. The experimental values are in reasonable agreement with the calculated values. In the case of β, we found that its variation with temperature below 4 k is caused by a phenomenon precursor to antiferromagnetism. Therefore the node type behaviour of the penetration depth has to be consistent with the precursor to antiferromagnetism.

The upper critical field can be obtained by a logarithmic plot of amplitude of the neutron diffraction. We have obtain the upper critical field of $TmNi_2B_2C$ shown in Fig.6.

The upper critical field is proportional to the superfluid density. It is highly anisotropic and characteristic of a node in the symmetry of the gap. At higher temperatures, it becomes comparatively isotropic as expected from the symmetry arguments. In Fig.7 we show the temperature dependence of the upper critical field. It is almost completely explained by the spin-orbit interaction [7]. We have seen that the apex angle β has zero value at two temperatures, the T_N and the T_c. Therefore it has a maximum value. This maximum value is also shown in Fig.7

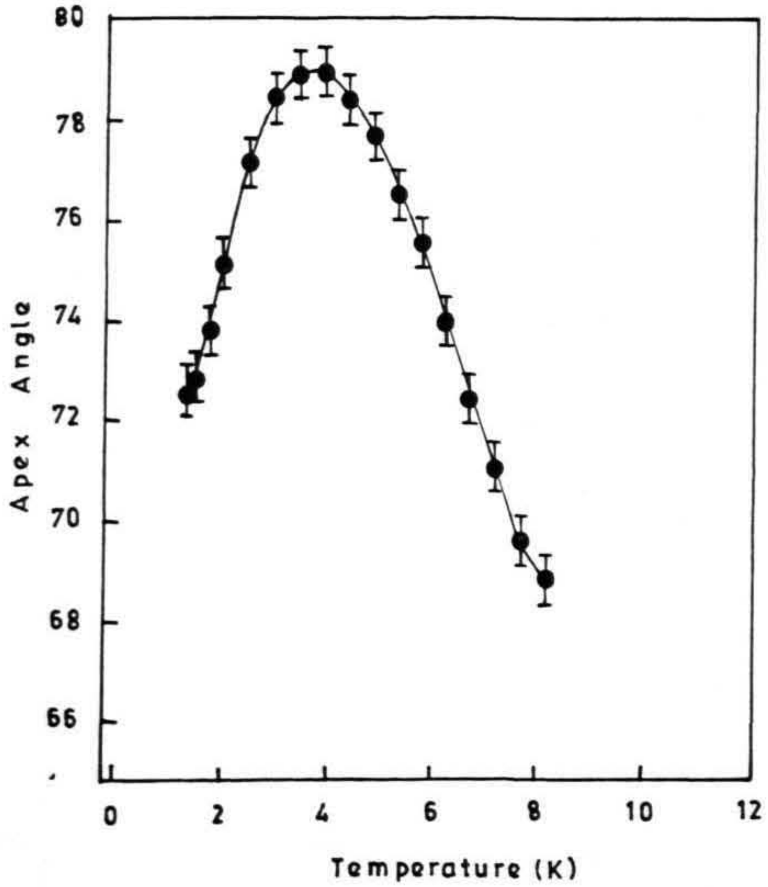

Fig.1 Apex angle of flux-line lattice in $TmNi_2B_2C$ as a function of temperature showing that it approaches T_N as well as T_c. The applied magnetic field is 3 K Oe.

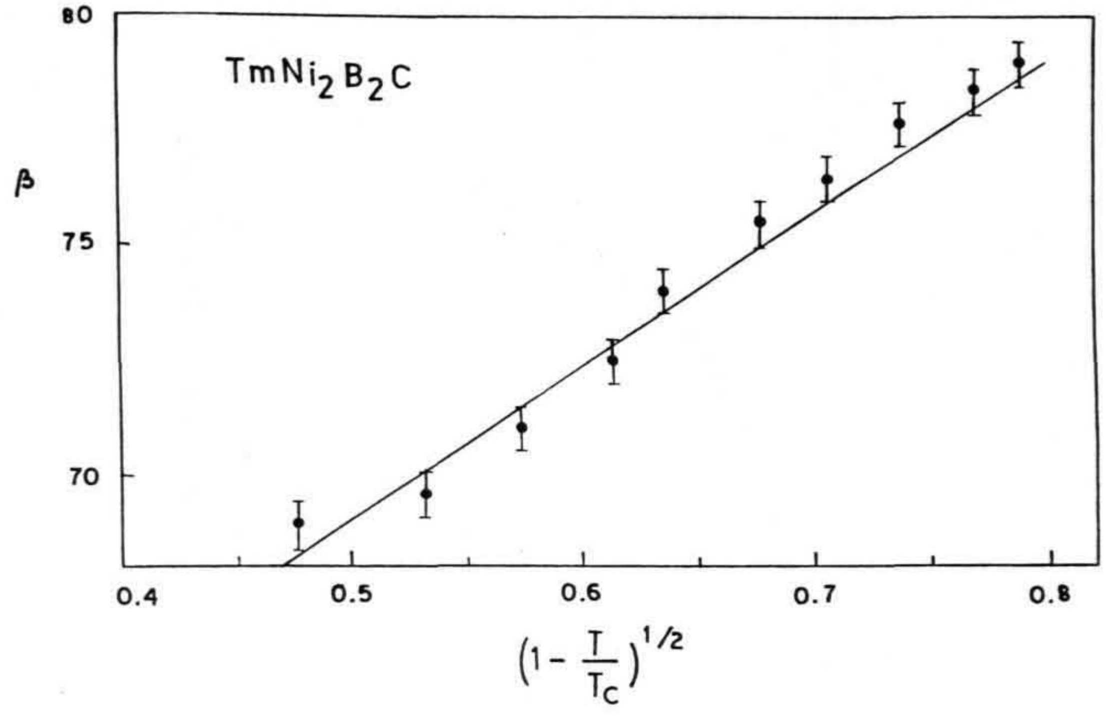

Fig.2 The apex angle for $10 > T > 4$ K as a function of $[1-(T/T_c)]^{1/2}$ showing soft vortex behaviour. This value is zero at T_c.

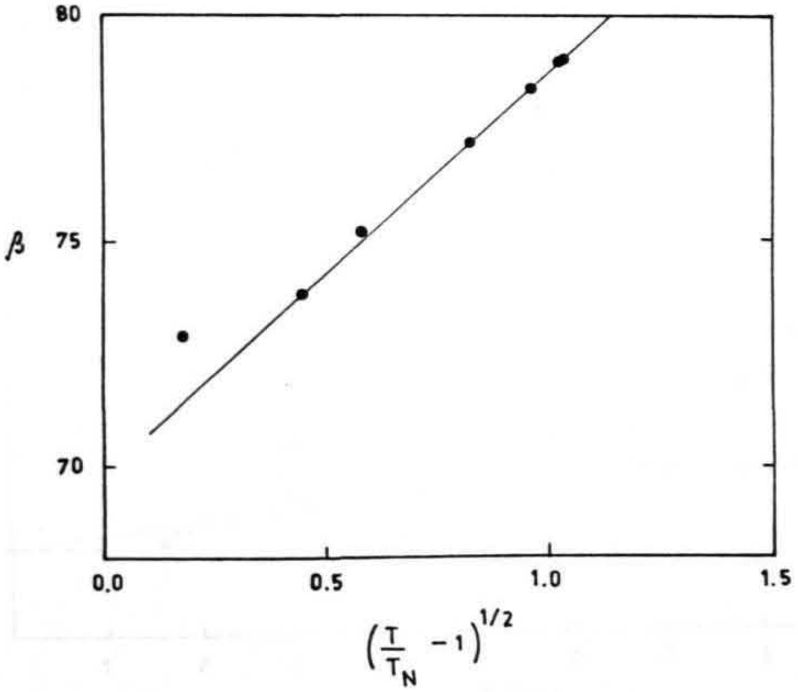

Fig.3 The apex angle for $1 < T < 4$ K showing soft vortex behaviour as a function of $[(T/T_N)-1]^{1/2}$ which approaches zero at T_N

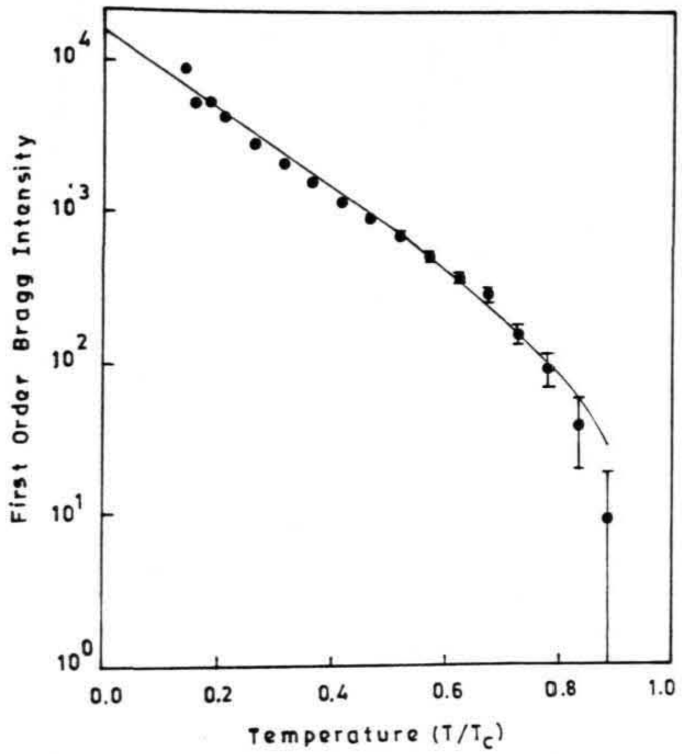

Fig.4 The first order Bragg neutron diffraction intensity as a function of temperature, vanishing at T_c

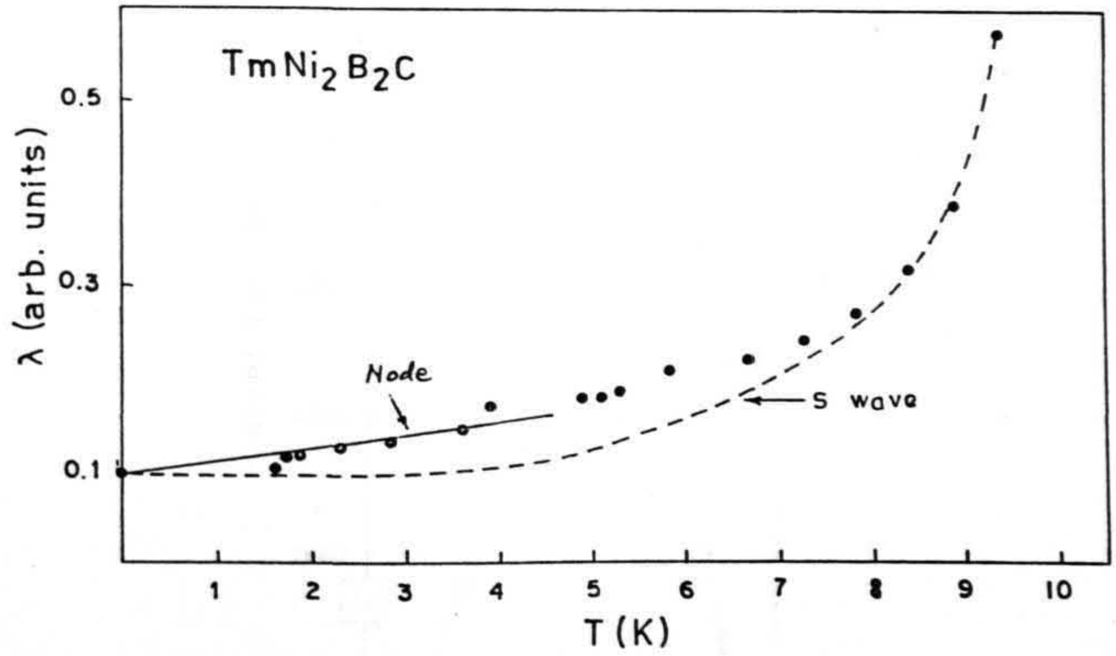

Fig.5 The London penetration depth obtained from the inverse fourth power of intensity as a function of temperature. The value calculated for the s-wave gap symmetry is shown by dashed line. The continuous line is when there is a node in the gap.

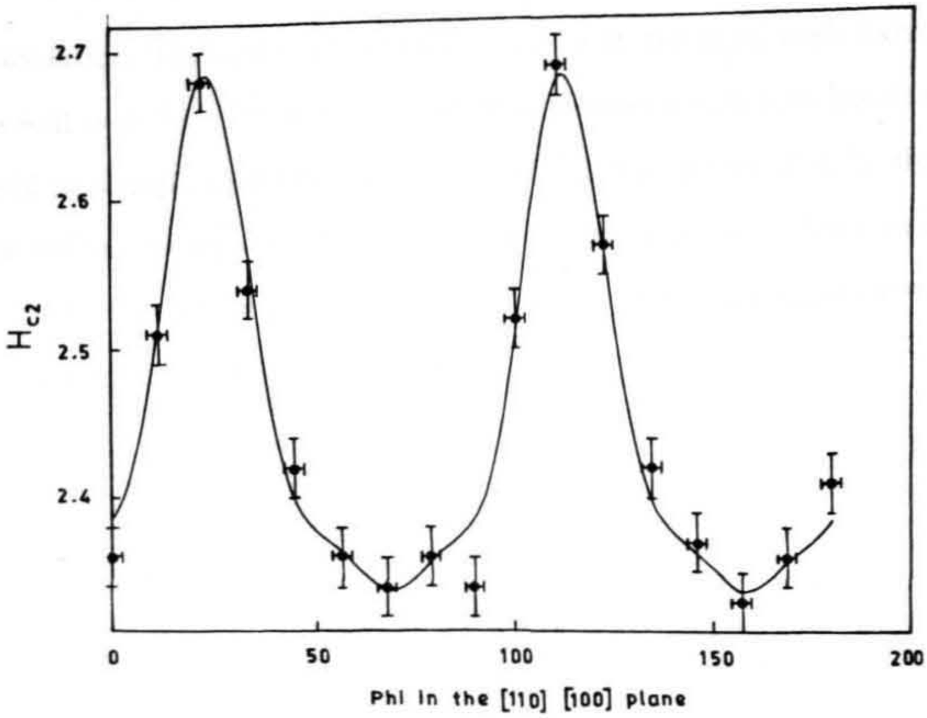

Fig.6 The angular dependence of the upper critical field of $TmNi_2B_2C$ at a temperature of 3 K showing nodes at low temperatures.

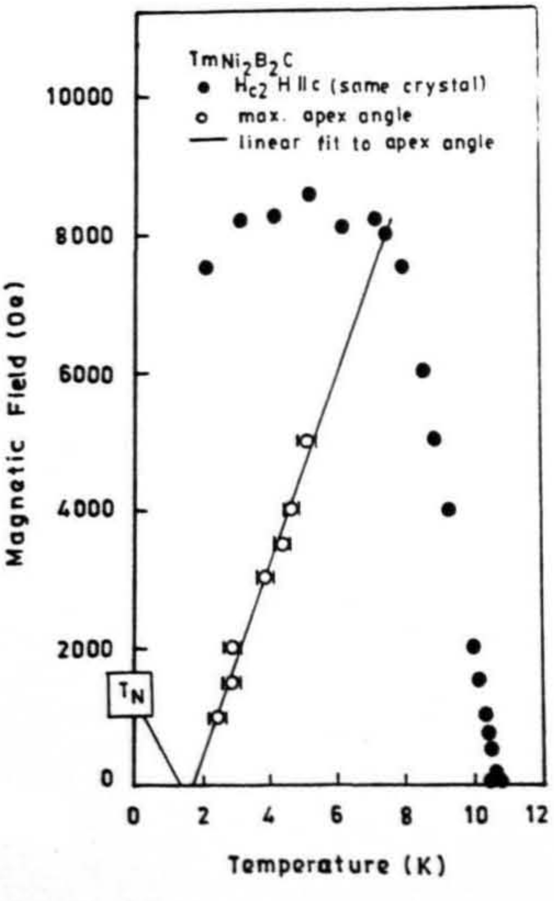

Fig.7 The upper critical field as a function of temperature. The maximum value of β is also shown.

In conclusion, $TmNi_2B_2C$ shows soft vortices with the apex angle having zero value at T_c as well as at T_N. The low temperature part shows a node type London penetration depth and anisotropic upper critical field. At high temperatures, $T < T_c$, the penetration depth as well as the upper critical field shows s-wave type dependence on temperature. Out of all the borocarbides, this compound has the largest temperature range between T_c and T_N due to the antiferromagnetism.

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Chapter-VII

Measurement of pinning frequency of a superconductor by a new method

The surface impedance of a system containing viscous damping and oscillating vortices is calculated, of which, in the limit of zero vortex mass, the real part is found to vary as $\omega^2(\omega^2+\omega_p^2)^{-1}$ where ω_p is the pinning frequency. For finite mass of a vortex, a characteristic frequency ω_r is found. For $\omega_p < \omega_r$ the real part of the resistivity shows peaks and for $\omega_p > \omega_r$ it oscillates. For zero mass of the vortex, the experimental measurements of the surface resistance of 2H-NbSe$_2$ are found to be in reasonable agreement with the theory which is used to measure the pinning frequency, $\omega_p \simeq 145.7$ MHz.

Introduction

Some time ago, it was shown by Stephen and Bardeen [1,2] that the viscosity of the vortices can give rise to a normal state conductivity, ρ_n^{-1}. The pinning frequency, ω_p is determined by the force constant of oscillations of a vortex and by the viscosity, which determines the force linear in the velocity,

$$\omega_p = \frac{k}{\eta} = \frac{k_0 \rho_n}{H_{c2}} j_c \tag{1}$$

the force on the vortex is kx with x as the displacement of the vortex. The periodic potential in the 'wash board' model is given by $V(x) = V_0[1 - \cos(k_0 x)]$ with $k_0 = 2\pi/r_p$ where r_p determines the characteristic length scale of the pinning interaction, usually taken as the lattice spacing [3]. For the flux line lattice (FLL) the wash board model gives the critical current as

$$j_c = \frac{k_0 V_0}{\phi_0} \tag{2}$$

where $\phi_0 = hc/2e$ is the unit flux. We have found [4] that large viscosity can cause a square root of magnetic field dependence in the penetration depth.

In this chapter we report that the resistivity depends on the frequency in such a way that the resistivity as a function of frequency can be used to measure the pinning frequency in the superconducting state of a type-II superconductor. We make use of the measured values [3] of resistivity as a function of frequency which are in reasonable agreement with the theory, leading to a measurement of $\omega_p \simeq 145.7$ MHz in 2H-NbSe$_2$. Due to finite mass of the vortex, for $\omega_p < \omega_r$, we predict that there are several peaks in the resistivity and for $\omega_p > \omega_r$ there are oscillations in the resistivity.

Theory

Recently, Volovik [5] has found that the mass of the vortex is given by

$$M \simeq m_e k_F^3 \xi^2 L \left(\frac{B_{c2}}{B}\right)^{\frac{1}{2}} \tag{3}$$

where m_e is the mass of the electron, k_F is the Fermi wave vector, ξ the coherence length, L the length of the current loop, B_{c2} the upper critical field inside the superconductor and B the magnetic field. The mass is obtained from the volume $\xi L R_v$ where R_v is the radius of the vortex in which the flux is quantized,

$$\pi R_v^2 B = n\phi_0 \tag{4}$$

so that $R_v = (n\phi_0/\pi B)^{1/2}$. The upper critical field is defined by $\pi\xi^2 B_{c2} = n\phi_0$ so that $R_v = \xi(B_{c2}/B)^{1/2}$ and $\xi R_v \simeq \xi^2(B_{c2}/B)^{1/2}$. The mass of the vortex is thus several hundred times the mass m_e of the electron. The equation (3) due to Volovik applies for the superclean limit of d-wave superconductors. On the other hand $NbSe_2$ is a conventional superconductor. Similar values were obtained by Suhl [6] who defined the inertial mass per unit length of a flux line of about $4000m_e$. If $\xi = 30\mathring{A}$ instead of $100\mathring{A}$, then the vortex mass is reduced to $444m_e$. Suhl mass is for a clean or dirty material and hence smaller than the superclean result by a factor $(k_F\xi)^2 \sim (\Delta/\epsilon_F)^2 \sim 10^4$. Thus while writing [7] the viscous and oscillatory forces, we include the Newtonian force on the vortex also so that

$$M\frac{dv}{dt} + \eta v + kx = \frac{1}{c}J\phi_0 \tag{5}$$

the intension being that there should be a mass term whether it is determined by Volovik or Suhl does not affect our equation. Therefore the vortex velocity is found to become

$$v = \frac{J\phi_0}{c\{\eta - i\omega M + (ik/\omega)\}} \tag{6}$$

The vortex moving with velocity v in a magnetic field B produces the electric field,

$$E_\varphi = -\frac{1}{c} vB \tag{7}$$

The London penetration depth is defined by the relation

$$\frac{dJ}{dt} = \frac{c^2}{4\pi\lambda_L^2}(E + E_\varphi) \tag{8}$$

where $E + \nabla\varphi = -\partial A/\partial t$. Substituting (6) in (7) and the resulting relation into (8) and the time dependence of the current as $J = J_0 e^{-i\omega t}$ we find

$$J = E\left[\frac{\phi_0 B}{c^2\{\eta - i\omega M + (ik/\omega)\}} - \frac{4\pi i\omega\lambda_L^2}{c^2}\right]^{-1} \tag{9}$$

which gives the resistivity as,

$$\rho = \frac{\phi_0 B\eta}{c^2(\eta^2 + m^2)} - i\left[\frac{m(\phi_0 B/c^2)}{\eta^2 + m^2} + \frac{4\pi\omega\lambda_L^2}{c^2}\right] \tag{10}$$

with $m = (k/\omega) - \omega M$. It may be noted that M is the vortex mass, m_e the electron mass and m is the quantity which has the dimensions of force constant devided by frequency which is the same as frequency multiplied by mass. The real part of resistivity is

$$Re\rho = \frac{\phi_0 B\eta\omega^2}{c^2[\omega^2\eta^2 + k^2 + \omega^4 M^2 - 2k\omega^2 M]} \tag{11}$$

For zero vortex mass, $M \simeq 0$, and the pinning frequency $\omega_p = k/\eta$, the above becomes

$$Re\rho = \frac{\phi_0 B\omega^2}{c^2\eta(\omega^2 + \omega_p^2)} \tag{12}$$

In dimensionless units, the resistivity is thus predicted to vary as

$$\frac{\rho(\omega)}{\rho(\omega = const)} = \frac{\omega^2}{\omega^2 + \omega_p^2} \tag{13}$$

Therefore a comparison of measured values of the resistivity as a function of frequency leads to a determination of ω_p.

The denominator of (11) is zero at the roots of the equation,

$$\frac{M^2}{\eta^2}\omega^4 + \omega^2(1 - 2\omega_p\frac{M}{\eta}) + \omega_p^2 = 0 \tag{14}$$

which are given by

$$\omega_{1,2}^2 = \frac{2\omega_p(M/\eta) - 1 \pm [(1 - 2\omega_p\frac{M}{\eta})^2 - 4\frac{M^2}{\eta^2}\omega_p^2]^{1/2}}{2\frac{M^2}{\eta^2}} \tag{15}$$

so that four peaks are predicted in the real part of the resistivity as a function of frequency. The roots are real only as long as,

$$\omega_p < \omega_r \tag{16}$$

where $\omega_r = \eta/(4M)$. When pinning frequency ω_p is less than ω_r, there are four real roots of (14) so that four peaks are predicted in the resistivity as a function of frequency. When $\omega_p > \omega_r$ the solutions become complex which means that the resistivity oscillates. The time of oscillations can be expressed as

$$\tau = \left[\frac{\tau_p^2 - (Im\tilde{\rho})\tau_p^2}{Im\tilde{\rho}}\right]^{1/2} \tag{17}$$

where $\omega_p = \tau_p^{-1}$ and $Im\tilde{\rho} = (Im\rho)(c^2\eta/\phi_0 B)$ which is described in dimensionless units. Thus at large frequencies, the resistivity shows new type of oscillations caused by the pinning and the vortex oscillations.

Comparison with experiments

The frequency dependence of the real part of the resistivity of 2H-NbSe$_2$ at a field of 0.5T and a temperature of 3K has been measured by Henderson et al. [3]. They have plotted the resistivity relative to the value at 2.8 GHz in dimensionless units as a function of logarithm of frequency of measurement from 10 MHz to 1000 MHz. According to our calculation the dimensionless resistivity varies as

$$\frac{r_s(\omega)}{r_s(2.8GHz)} = \frac{\omega^2}{(\omega^2 + \omega_p^2)} \tag{18}$$

without the logarithmic divergence at $\omega = 0$. In Fig.1 we have plotted the dimensionless resistivity as a function of frequency from equation (18) which does not have any logarithmic term. The calculated values for $\omega_p = 145.7$ MHz, 212.7 MHz, and 327 MHz are shown as there is no other parameter. At low frequencies, the calculated curve with $\omega_p = 145.7$ MHz fits the data very well. At higher frequencies there are deviations from the predicted value. Thus we have obtained a new method of measuring the pinning frequency. The expression (18) is independent of the vortex mass.

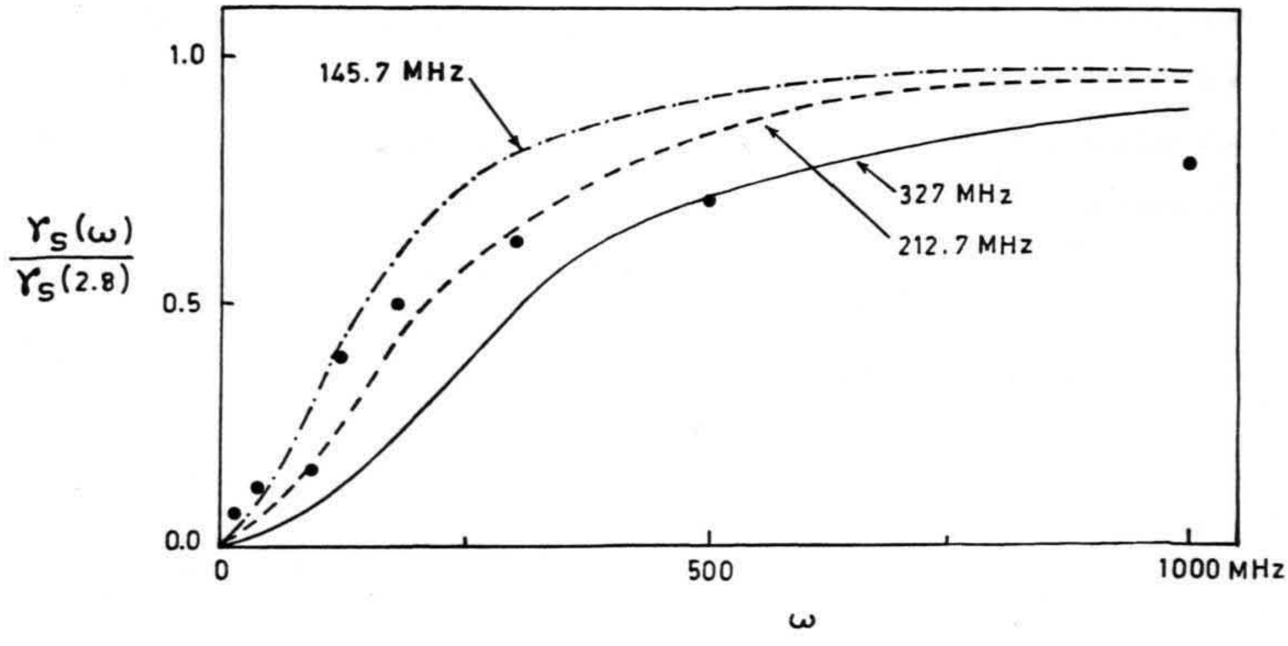

Fig.1 The relative surface resistivity of 2H-NbSe$_2$ as a function of frequency. The dots are taken from the experimental measurements performed by Henderson et al. [3]. The calculated curves are shown for three different values of the pinning frequency, ω_p. At low frequencies the curve calculated for $\omega_p \simeq 145.7$ MHz agrees with the experimental measurements.

New effects are predicted at very small magnetic fields. In particular it is found that the resistivity oscillates if the field is less than $16(m_e k k_F^3 \xi^2 L)^2 B_{c2}/\eta^4$. It may be noted that only the frequency is being varied to measure the pinning frequency but the temperature is kept constant. The symmetry of the system may change from s wave to d wave in varying the temperature and the flux lattice may melt [8,9].

Conclusions

The vortex oscillations, viscosity and their kinetic energy all contribute to the current. When the contribution of the kinetic energy is small, the measurement of resistivity leads to the evaluation of the pinning frequency. Our theoretical expression for the resistivity is compared with the experimental measurements of the resistivity of 2H-$\mathrm{NbSe_2}$ as a function of frequency which gives for the pinning frequency, $\omega_p \simeq 145.7$MHz. For finite mass of a vortex interesting effects are predicted at small fields.

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